Coping with singularities in the design of parallel manipulators

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Presentation outline

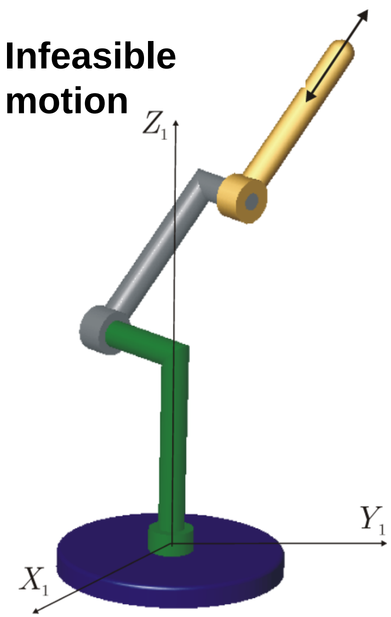
What is a singularity?

How to find them?

How to cope with them: elimination, avoidance, …

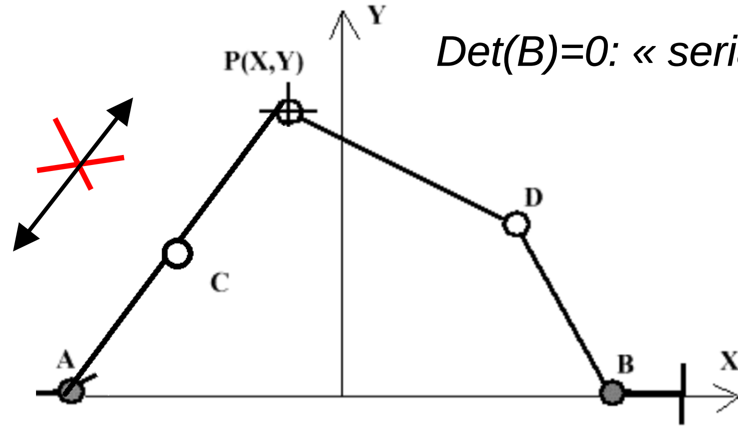
Case studies - examples

Common singularities (Gosselin, 1990)



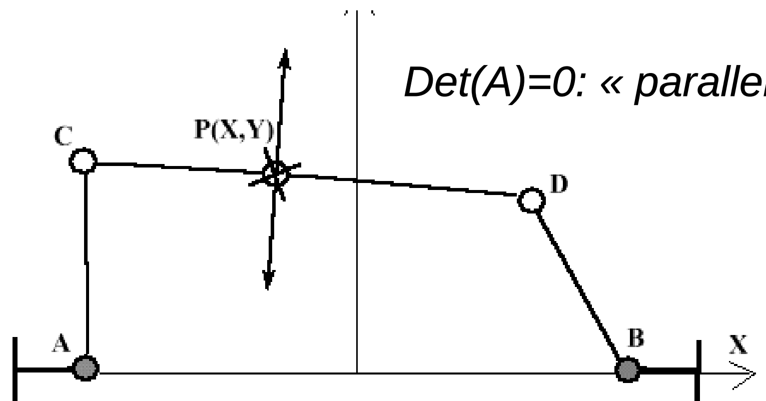
Serial robot

Det(B)=0: « serial, type 1 singularity »



$$A\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix} + B\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = 0$$

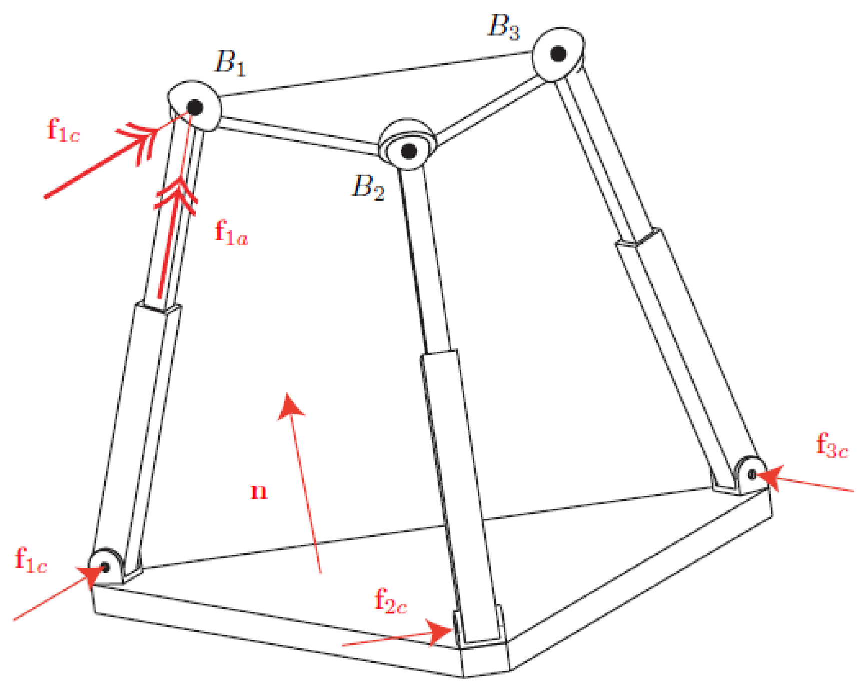
Det(A)=0: « parallel , type 2 singularity »



Parallel robot

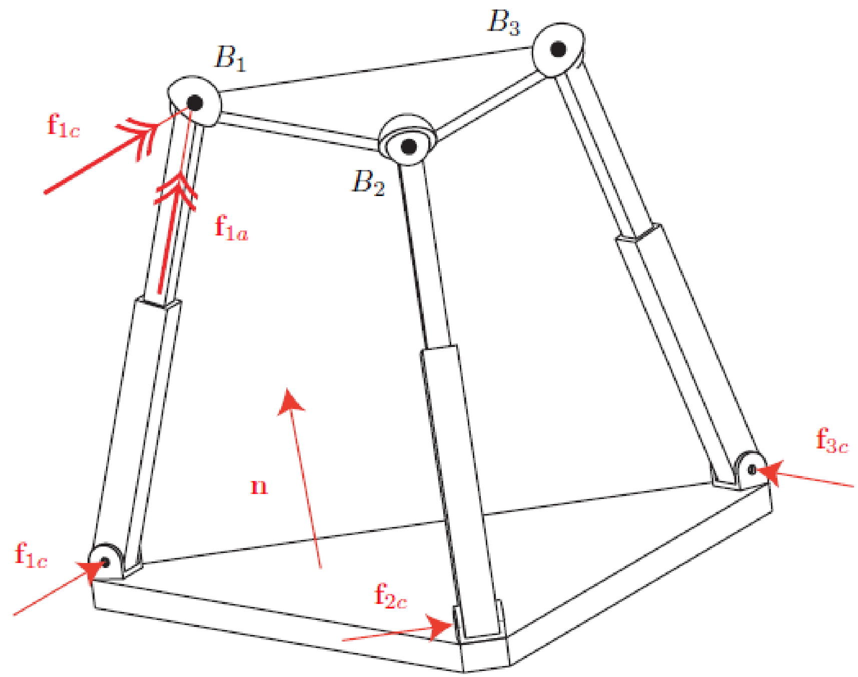
Constraint singularities (CS)

- CS exist only in lower-mobility parallel manipulators (with less than 6-DOF)
- For such manipulators, the platform motion is not solely constrained by the actuators, but also by the special leg arrangement
- A CS occurs when the system of leg constraints degenerates



Constraint singularities (CS)

- The 3-RPS robot:
- Leg constraints = 3 forces parallel to the base plane and passing through Bi
- Platform mobility = one vertical translation + 2 (non pure) rotations



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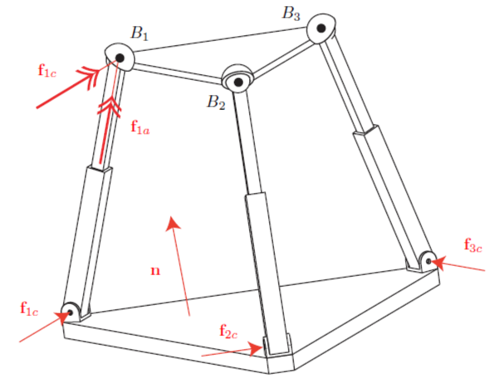
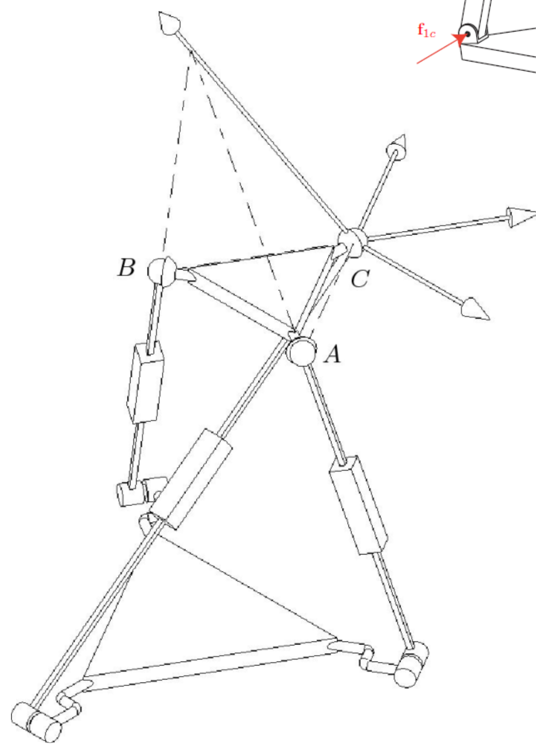
Constraint singularities (CS)



- The 3-RPS robot
- Leg constraints = 3 forces parallel to the base plane and passing through Bi
- Platform mobility = one vertical translation + 2 (non pure) rotations
- CS: the 3 contraint forces intersect => a third rotation is gained



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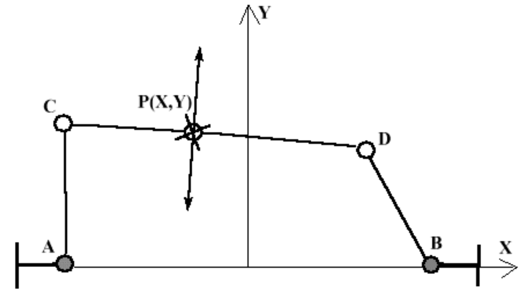
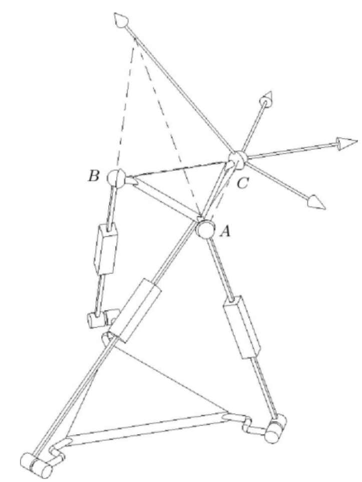
Actuation singularity vs Constraint singularity



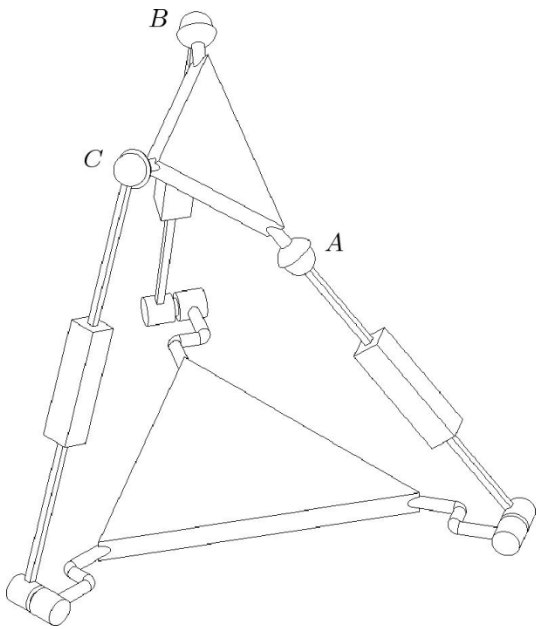
$$A\dot{q} + Bv = 0$$

- Actuation singularity: one of the DOF of the platform cannot be controlled by the actuators. Can be avoided by a change in the actuated joints. Can be detected with the input-output velocity equations by det(A)=0

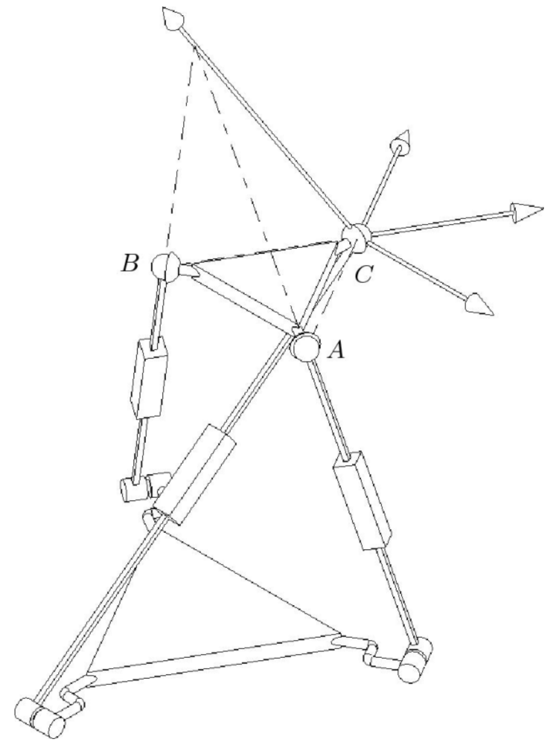
- Constraint singularity: the platform acquires a new DOF. Remains if actuated joints are changed. Not detected by det(A)=0 with the input/output equations.



Actuation singularity vs Constraint singularity



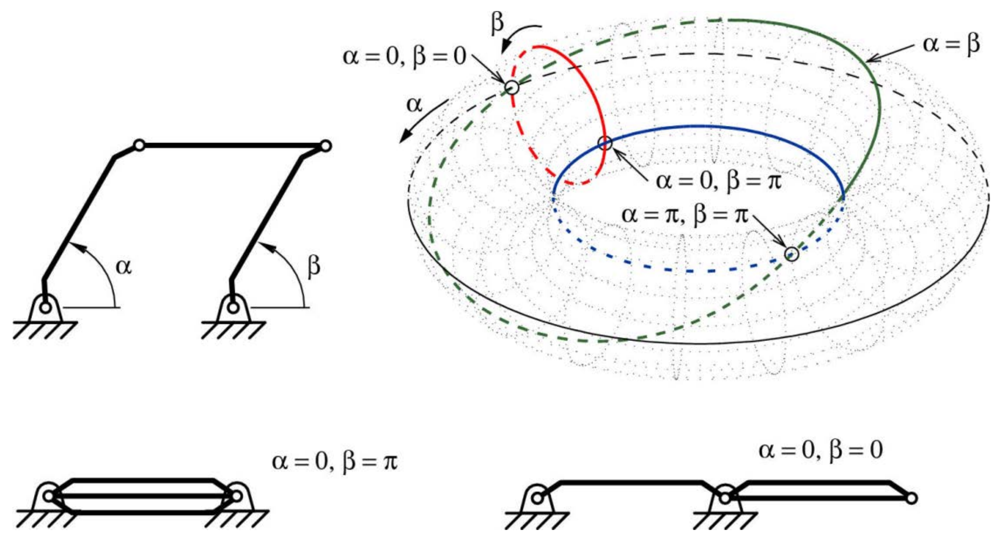
A 3-RPS robot in an Actuation Singularity: infinitesimal rotation about BC with locked actuators



A 3-RPS robot in a Constraint Singularity: one more rotation is gained about C

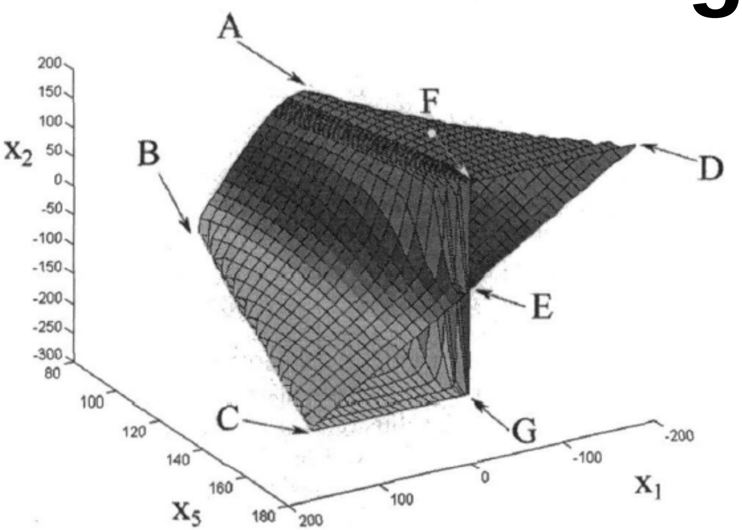
Constraint singularities and operation modes

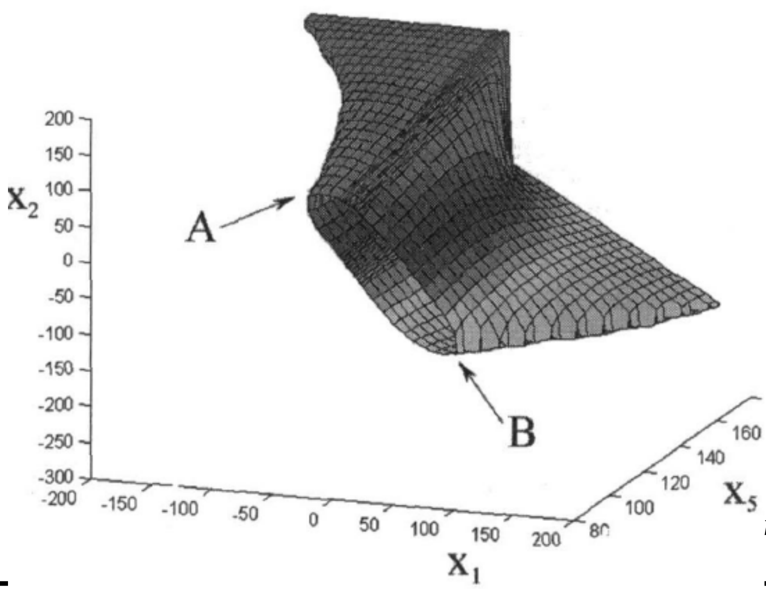
- A constraint singularity is associated with a singularity of the C-space (configuration space)
- Near a constraint singularity, there are different C-space submanifolds
- A constraint singularity can be regarded as a branching point between different types of motions



Parallelogram and antiparallelogram

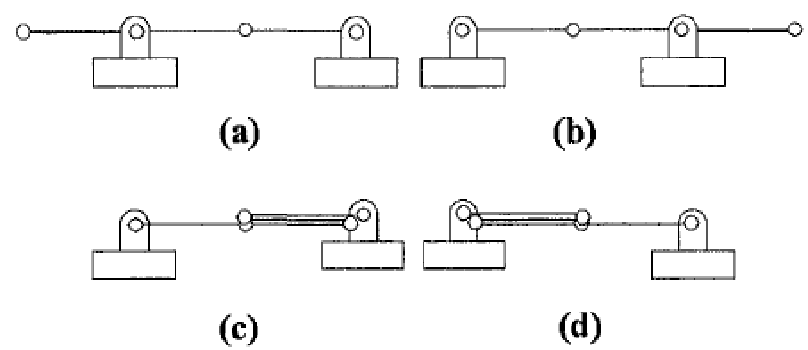
Constraint singularities and operation modes





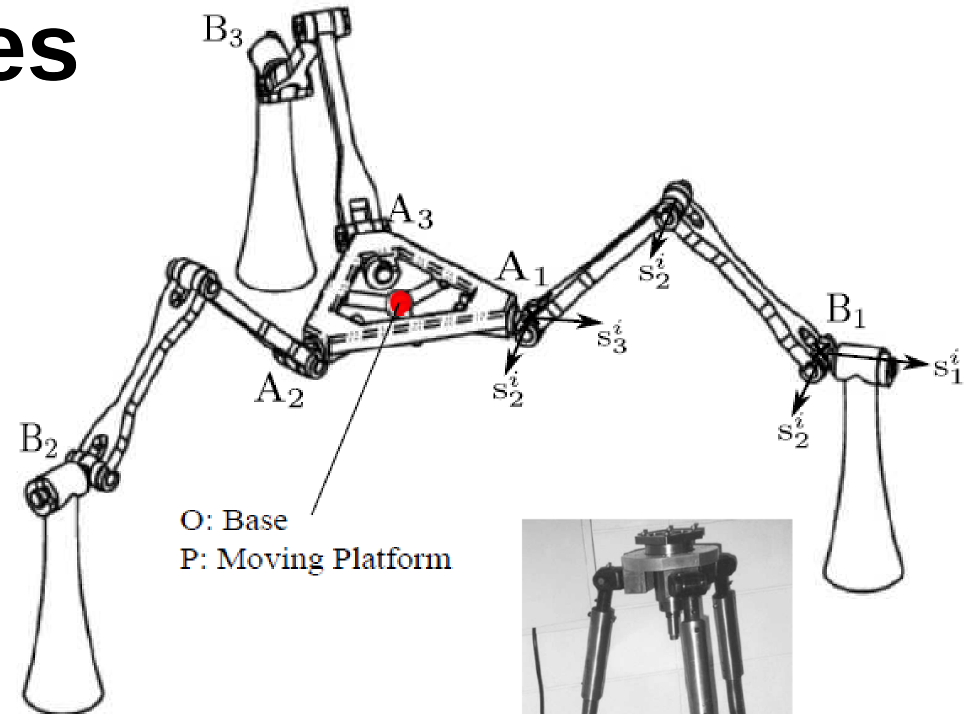
The 5-bar mechanism and its C-space singularities:

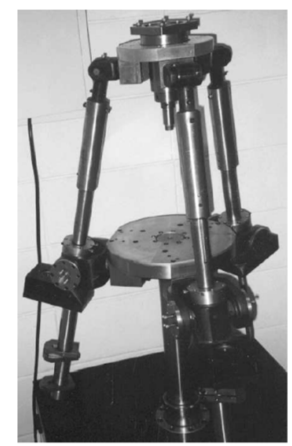
- (a) ⇔ C and D
- (d) ⇔ G and F
- (c) ⇔ E
- (b) ⇔ F (not shown)



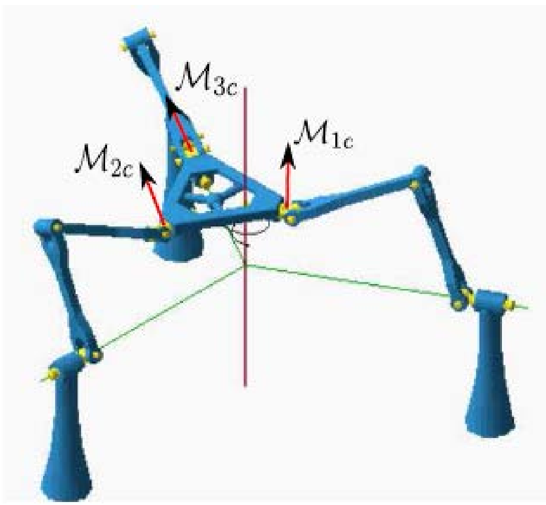
Constraint singularities and operation modes

- A parallel manipulator with multiple operation modes: the DYMO 3-URU *(Zlatanov et al., ARK'2002)*.
- This PM has 7 distinct operation modes
- Note: same results with the SNU 3-UPU (*Walter et al., Contemporary Mathematics, Vol 496, 2009*),

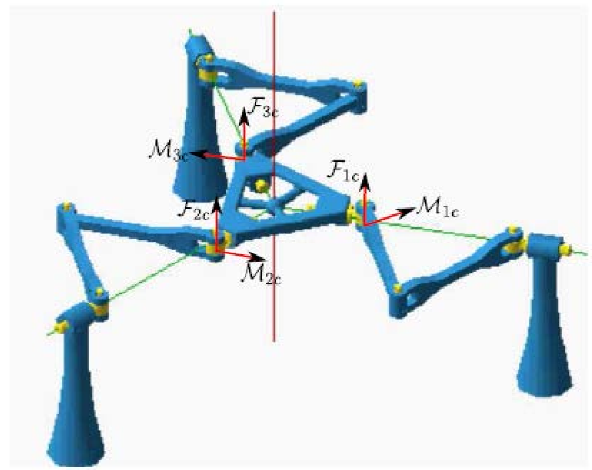




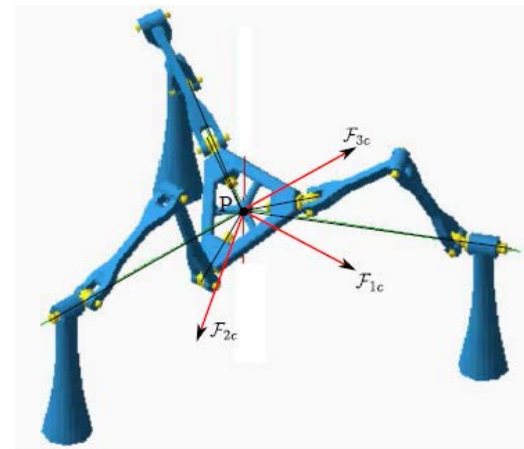
Constraint singularities and operation modes



Translation mode



Planar mode

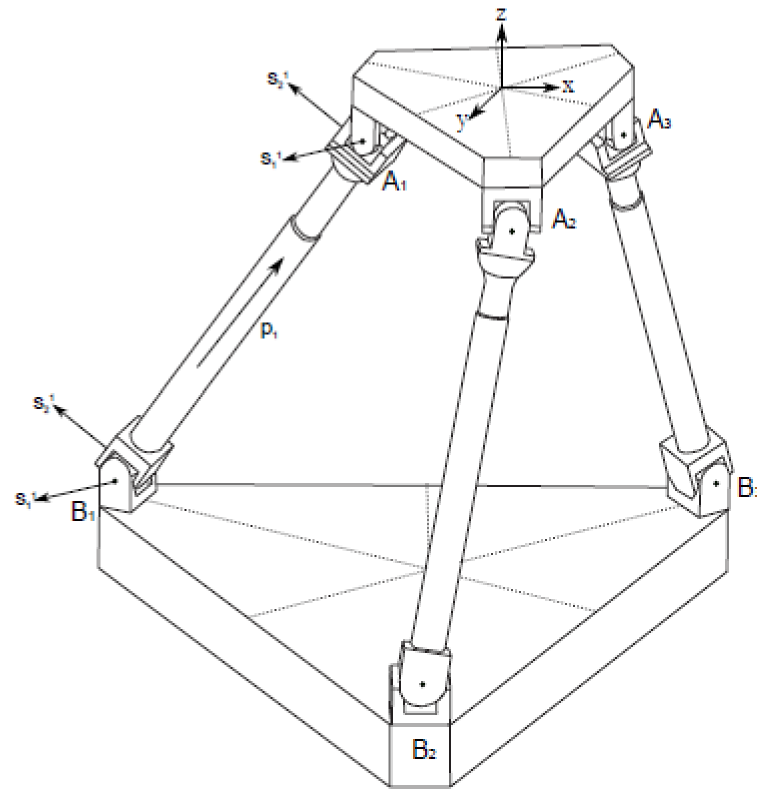


Spherical mode

Constraint singularities and operation modes

- The TSAI 3-UPU manipulator has also several distinct operation modes

- In addition to the translational mode shown on the right (this robot was designed to have this motion!), it can exhibit 4 more operation modes



The Tsai 3-UPU manipulator in its translational mode

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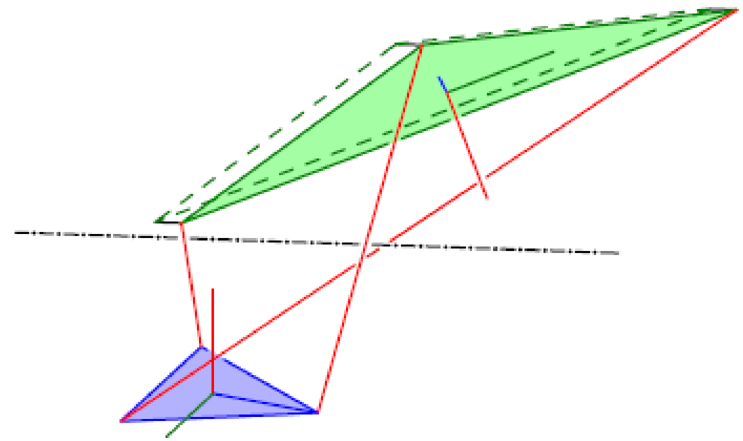
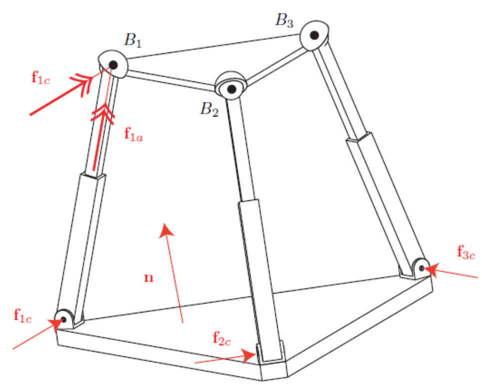
Constraint singularities and operation modes

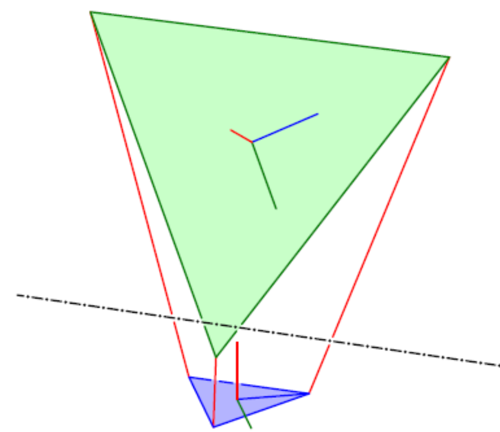


- The 3-R<u>P</u>S parallel manipulator has 2 operation modes but they are more difficult to identify
- Both operation modes are defined by a vertical translation and two non pure rotations

Husty, M., Schadlbauer, J., Caro, S., and Wenger, P., 2013, “Self-Motions of 3-RPS Manipulators”, Frontiers of Mechanical Engineering, Vol. 8(1), pp. 62–69, DOI 10.1007/s11465-013-0366-3

Schadlbauer, J., Nurahmi, L., Husty, M., Wenger, P. and Caro, S., “Operation Modes in Lower-Mobility Parallel Manipulators”, Second Conference on Interdisciplinary Applications of Kinematics, Lima, Peru,





Constraint singularities and operation modes

- This 3-PRS parallel manipulator has 2 operation modes (same as for the 3-RPS) but their designer were not aware of this fact
- In which operation mode does the Sprint Z3 operate?
- Would the other operation mode be more suitable for milling?

The Sprint Z3 tool-head by DS-Technology

Constraint singularities and operation modes

- How to cope with these features in the design?
- It is of primary importance to detect the constraint singularities at the design stage
- Existence of several operation modes might be of interest (reconfigurable manipulator)
- But could be a problem if not anticipated… (see the SNU 3-UPU manipulator)

Leg singularities

- Not always detected buy det(B) with the input/output equations
- Example: the Orthoglide, the Tripteron

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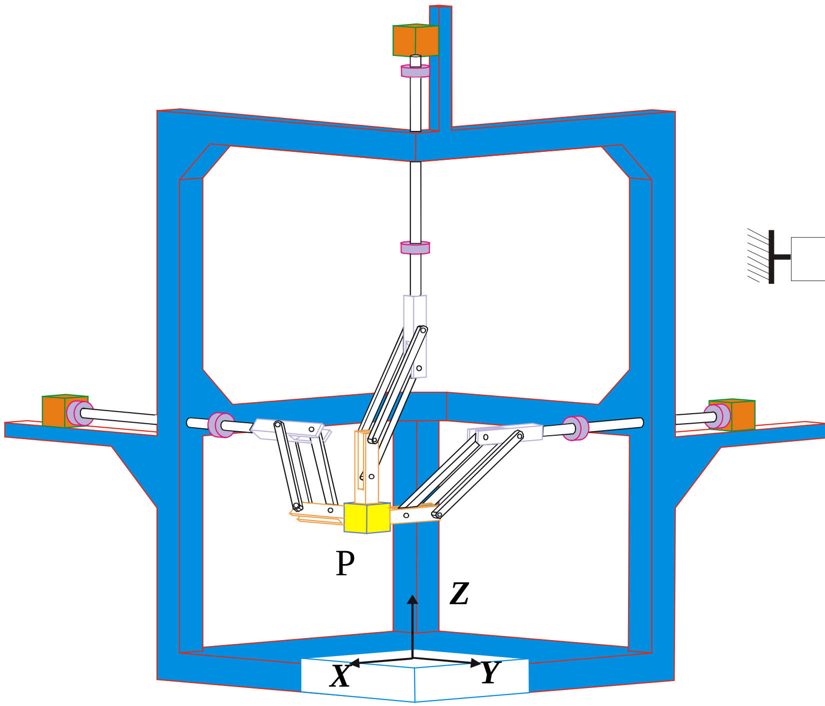
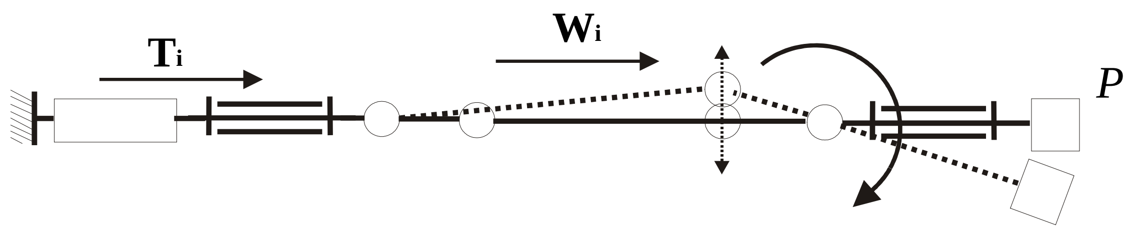
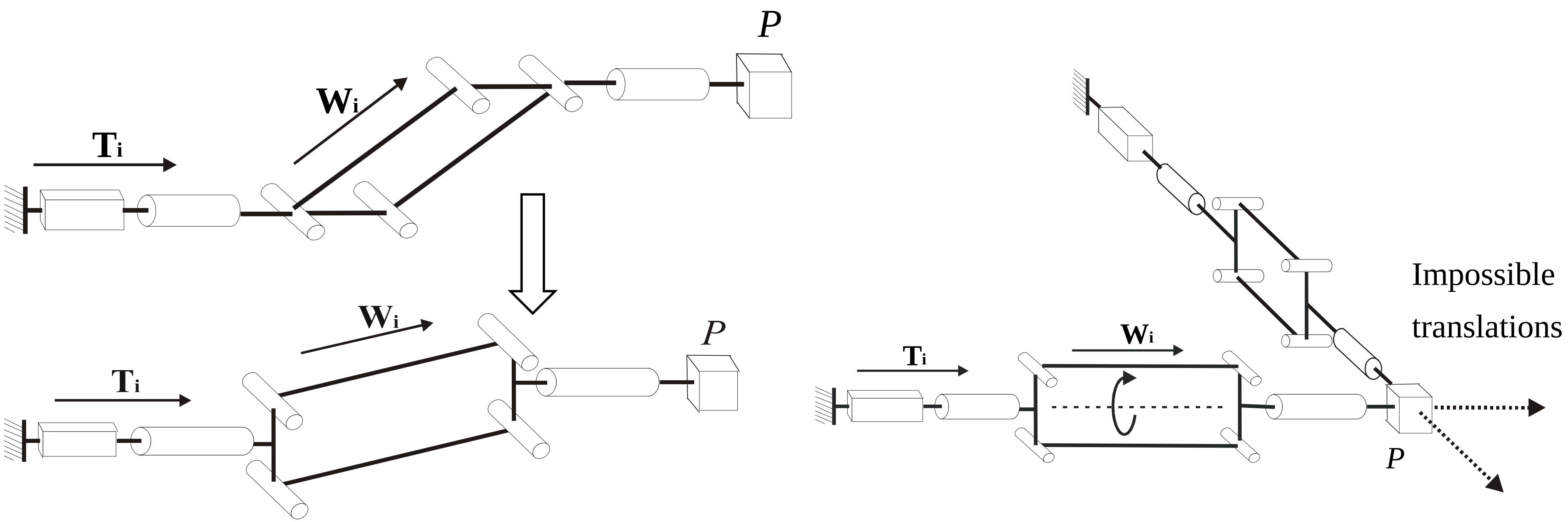
Leg singularity in the first Orthoglide version





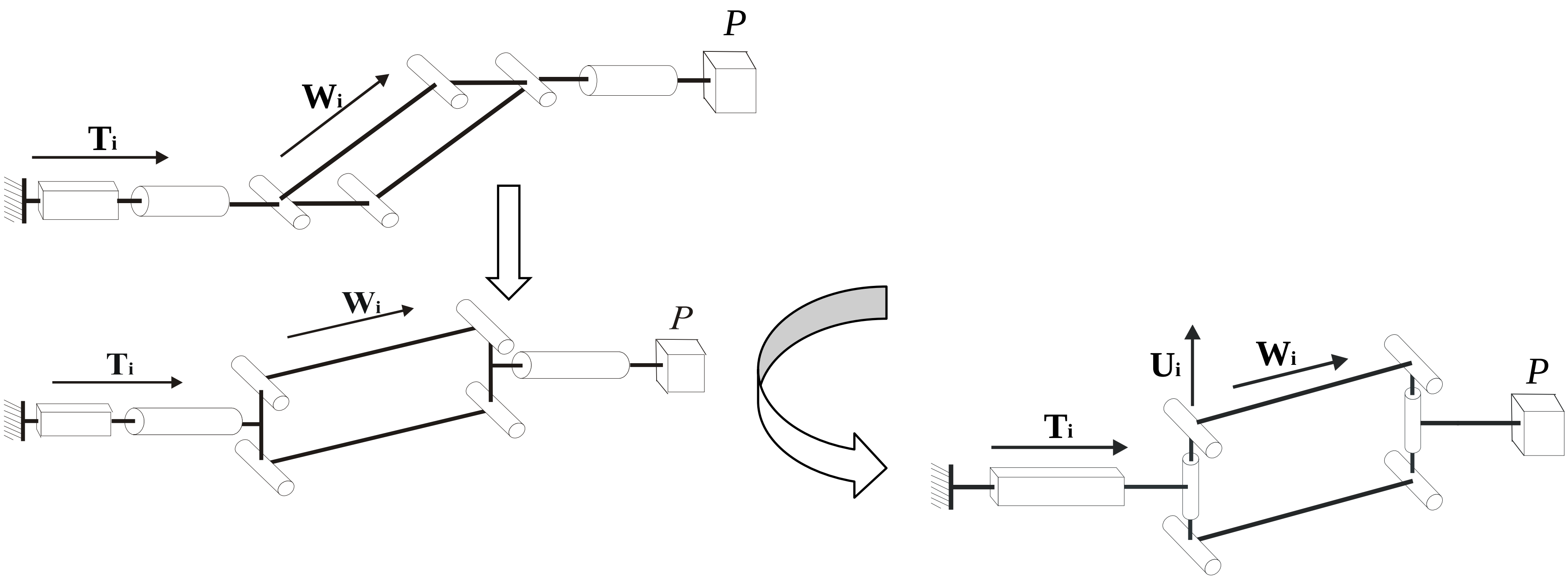
In the configuration where both jacobian matrices A and B are isotropic (the 3 legs are orthogonal), the parallelograms are in the flat singular configuration!!

Another leg singularity…



A Redundant-Passive-Motion (RPM), Impossible Input (II), Impossible Output (IO) type singularity (det(A) ≠ 0, det(B) ≠ 0)

Leg singularity



This last leg arrangement removes the leg singularity in the isotropic configuration

Finding the singularities?

- Determine the actuation and wrench systems using screw theory
- Write the expression of the full Jacobian matrix:

(n<6)-*dof* Parallel Manipulator

- Actuators apply a n-actuation wrench system $\mathcal{W}_a$
- Limbs apply a $(6-n)$-constraint wrench system $\mathcal{W}_c$

$$\mathbf{J}_E^T = [\underbrace{\hat{\$}_a^1 \; \ldots \; \hat{\$}_a^n}_{\text{a basis of } \mathcal{W}^a} \; \underbrace{\hat{\$}_c^1 \; \ldots \; \hat{\$}_c^{6-n}}_{\text{a basis of } \mathcal{W}^c}]$$

Finding the singularities?

- Usual method: derive det(J_E)
- Often leads to very large expressions with no insight in the geometric condition
- One possibility: use Grassman-Cayley Algebra (Ben-Horin et al, Ph.D thesis, 2008), together with the wrench graph (Amine, Ph.D thesis, 2011)
- Makes it possible to work at symbolic level where points and lines are represented in a coordinate-free form (superbracket expression)

Finding the singularities?

$$\mathbf{J}_E^T = [\underbrace{\hat{\$}_a^1 \ \ldots \ \hat{\$}_a^n}_{\text{a basis of } \mathcal{W}^a} \ \underbrace{\hat{\$}_c^1 \ \ldots \ \hat{\$}_c^{6-n}}_{\text{a basis of } \mathcal{W}^c}]$$

$$\downarrow$$

Superbracket $S = [\mathtt{ab\ cd\ ef\ gh\ ij\ kl}] = \sum_{i=1}^{24} y_i$

$$\downarrow$$

$$y_1 = [\mathtt{abcd}][\mathtt{efgi}][\mathtt{hjkl}] \qquad y_2 = -[\mathtt{abcd}][\mathtt{efhi}][\mathtt{gjkl}]$$

$$\vdots$$

$$y_{23} = [\mathtt{abch}][\mathtt{defj}][\mathtt{gikl}] \qquad y_{24} = -[\mathtt{abdh}][\mathtt{cefj}][\mathtt{gikl}]$$

Finding the singularities?

- If the points are chosen judiciously on the 3 lines composing J_E, many brackets vanish and the expression of $\det(J_E)$ simplifies

- A bracket, [a b c d] vanishes whenever :
  1. two points are repeated
  2. three points are collinear
  3. the four points are coplanar

Amine, S., Tale-Masouleh, M., Caro, S., Wenger, P., and Gosselin, C., 2012, "Singularity Conditions of 3T1R Parallel Manipulators with Identical Limb Structures", ASME Journal of Mechanisms and Robotics, Vol. 4(1), pp. 011011-1–011011-11, doi :10.1115/1.4005336. hal-00642238

Amine, S., Caro, S., Wenger, P. and Kanaan, D., 2012, "Singularity Analysis of the H4 Robot using Grassmann-Cayley Algebra", Robotica, Vol. 30(7), pp. 1109–1118, doi :10.1017/S0263574711001330. hal-00642230

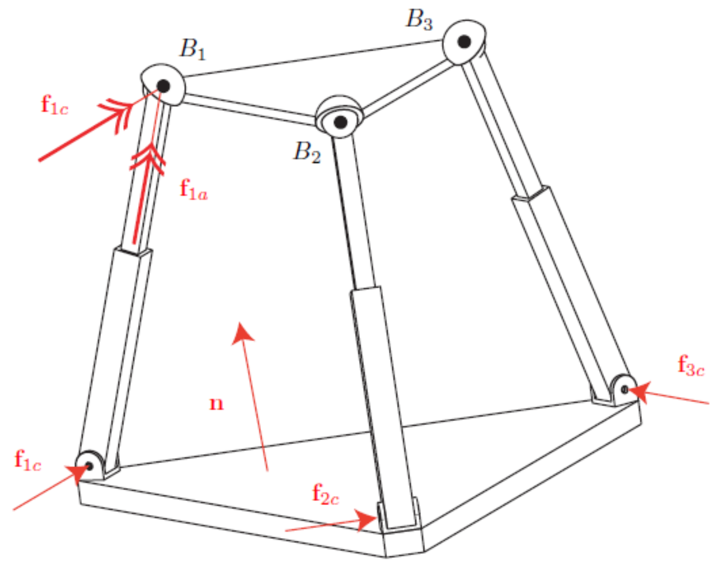
Finding the singularities?

- To simplify: reduce the number of points, use collinear points, coplanar points and points at infinity
- To avoid difficult hand calculations: a graphical interface with automatic calculation and geometric interpretation has been developed (Ben-Horin, Ph.D thesis 2008, Amine, Ph.D thesis 2011)

Caro, S., Nurahmi, L. and Wenger, P., "Graphical User Interface for the Singularity Analysis of Lower-Mobility Parallel Manipulators", 5th European Conference on Mechanism Science. Guimarães, Portugal, September 16–20, 2014.

Finding the singularities?

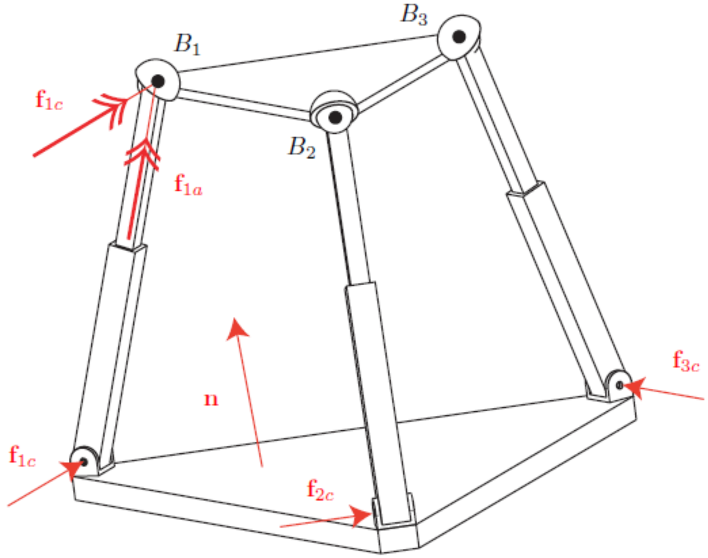
- Example: the 3-RPS manipulator

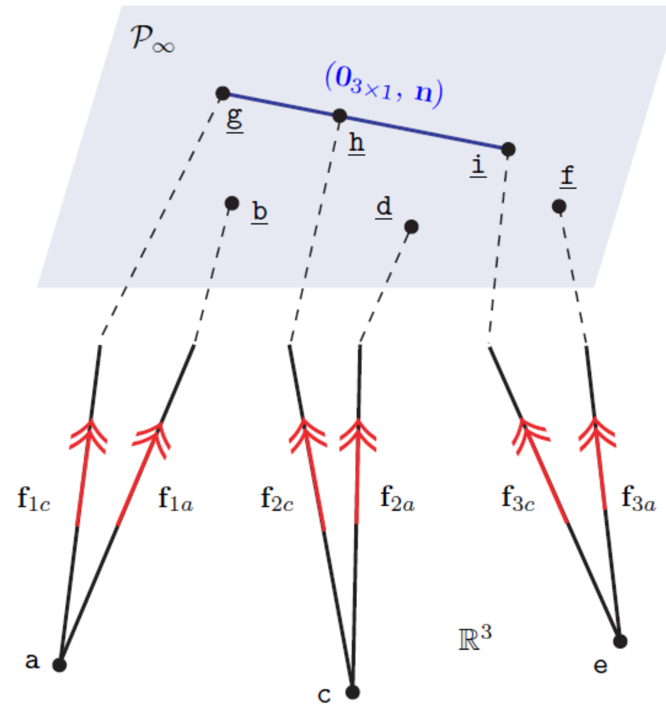


$$
\begin{aligned}
\mathbf{J}_E^T &= \begin{bmatrix} \hat{\mathcal{F}}_{1a} & \hat{\mathcal{F}}_{2a} & \hat{\mathcal{F}}_{3a} & \hat{\mathcal{F}}_{1c} & \hat{\mathcal{F}}_{2c} & \hat{\mathcal{F}}_{3c} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{f}_{1a} & \mathbf{f}_{2a} & \mathbf{f}_{3a} & \mathbf{f}_{1c} & \mathbf{f}_{2c} & \mathbf{f}_{3c} \\ \mathbf{r}_{B_1} \times \mathbf{f}_{1a} & \mathbf{r}_{B_2} \times \mathbf{f}_{2a} & \mathbf{r}_{B_3} \times \mathbf{f}_{3a} & \mathbf{r}_{B_1} \times \mathbf{f}_{1c} & \mathbf{r}_{B_2} \times \mathbf{f}_{2c} & \mathbf{r}_{B_3} \times \mathbf{f}_{3c} \end{bmatrix}
\end{aligned}
$$

Finding the singularities?

- Example: the 3-R$\underline{\text{P}}$S manipulator





- The superbracket expression simplifies to:

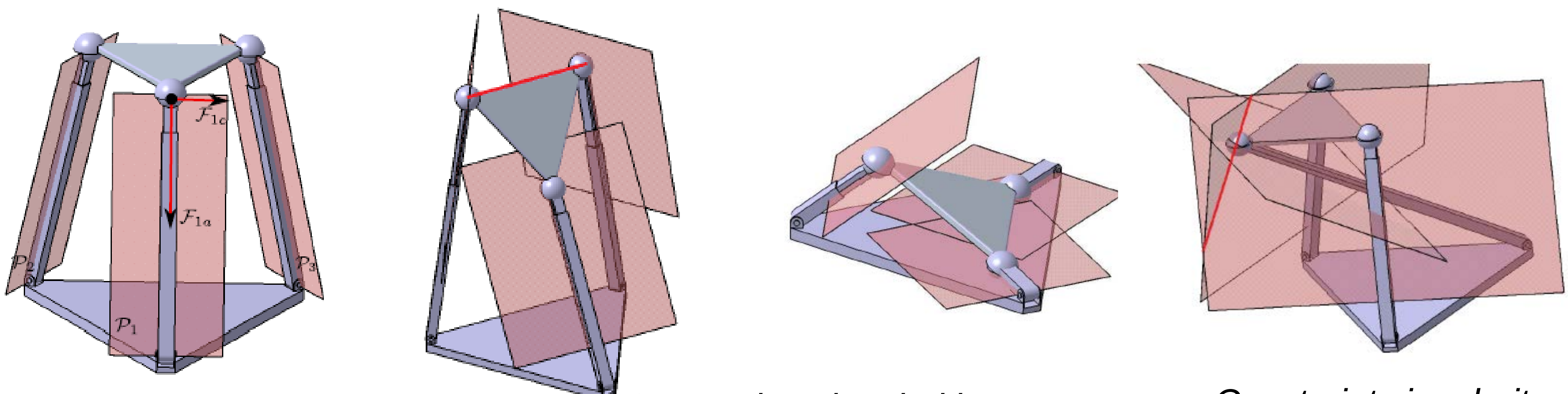
$$S = [a\underline{b}\ a\underline{g}\ c\underline{d}\ c\underline{h}\ e\underline{f}\ \underline{i}] = [ac\dot{\underline{h}}e][a\underline{b}\underline{g}\dot{c}][\dot{\underline{d}}\underline{f}e\underline{i}]$$

Finding the singularities?

- Ex: the 3-R$\underline{\text{P}}$S. The superbracket expression simplifies to:

$$S = [a\underline{b}\ a\underline{g}\ c\underline{d}\ c\underline{h}\ e\underline{f}\ \underline{i}] = [ac\underline{\dot{h}}e][a\underline{b}\underline{g}\dot{c}][\underline{\dot{d}}fe\underline{i}]$$

The singularity condition of 3-R$\underline{\text{P}}$S occurs whenever the four foregoing planes (ace), (a$\underline{b}\underline{g}$), ($\underline{f}e\underline{i}$), and ($\underline{h}c\underline{d}$) intersect at least at one point.



2 actuation singularities *Constraint singularity*

Finding the operation modes?

- Constraint singularity => transition between two operation modes
- How to find the distinct operation modes?
- Screw theory gives a local analysis = differential mobility in a given configuration (in a singular or regular one)
- Operation modes are encoded in the C-space => they should be obtained from the constraint equations that define the C-space

Finding the operation modes?

- Write all constraint equations in polynomial form. The vanishing set of the ideal generated by this set of equations defines a variety, which is nothing else than the C-space
- The primary decomposition of this ideal gives the subvarieties that constitute the C-space, namely, the operation modes (Husty et al., Robotica, 2007)
- Ex: the 3-RPS manipulator

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Finding the operation modes?

- Ex: the 3-RPS manipulator. Constraint equations (with dual quaternions):

$$g_1 : x_0 x_1 = 0$$

$$g_2 : h_2 x_2^2 - h_2 x_3^2 - 2x_0 y_3 - 2x_1 y_2 + 2x_2 y_1 + 2x_3 y_0 = 0$$

$$g_3 : 2h_2 x_0 x_1 + h_2 x_2 x_3 - x_0 y_2 + x_1 y_3 + x_2 y_0 - x_3 y_1 = 0.$$

$$\begin{aligned} g_4 :&(h_1 - h_2)^2 x_0^2 + (h_1 + h_2)^2 x_1^2 + (h_1 + h_2)^2 x_2^2 + (h_1 - h_2)^2 x_3^2 \\ &+ 4(h_1 - h_2) x_0 y_3 + 4(h_1 + h_2) x_1 y_2 - 4(h_1 + h_2) x_2 y_1 \\ &- 4(h_1 - h_2) x_3 y_0 + 4(y_0^2 + y_1^2 + y_2^2 + y_3^2) - (x_0^2 + x_1^2 + x_2^2 + x_3^2) r_1^2 = 0 \end{aligned}$$

$$\begin{aligned} g_5 :&(h_1 - h_2)^2 x_0^2 + (h_1 + h_2)^2 x_1^2 + (h_1^2 + h_2^2 - h_1 h_2) x_2^2 + (h_1^2 + h_2^2 + h_1 h_2) x_3^2 - 2(h_1 \\ &- h_2) x_0 y_3 - 2(h_1 + h_2) x_1 y_2 + 2(h_1 + h_2) x_2 y_1 + 2(h_1 - h_2) x_3 y_0 - 2\sqrt{3}(h_1 \\ &- h_2) x_0 y_2 + 2\sqrt{3}(h_1 + h_2) x_1 y_3 + 2\sqrt{3}(h_1 - h_2) x_2 y_0 - 2\sqrt{3}(h_1 + h_2) x_3 y_1 \\ &- 2\sqrt{3} h_1 h_2 x_2 x_3 + 4(y_0^2 + y_1^2 + y_2^2 + y_3^2) - (x_0^2 + x_1^2 + x_2^2 + x_3^2) r_2^2 = 0 \end{aligned}$$

$$\begin{aligned} g_6 :&(h_1 - h_2)^2 x_0^2 + (h_1 + h_2)^2 x_1^2 + (h_1^2 + h_2^2 - h_1 h_2) x_2^2 + (h_1^2 + h_2^2 + h_1 h_2) x_3^2 - 2(h_1 \\ &- h_2) x_0 y_3 - 2(h_1 + h_2) x_1 y_2 + 2(h_1 + h_2) x_2 y_1 + 2(h_1 - h_2) x_3 y_0 + 2\sqrt{3}(h_1 \\ &- h_2) x_0 y_2 - 2\sqrt{3}(h_1 + h_2) x_1 y_3 - 2\sqrt{3}(h_1 - h_2) x_2 y_0 + 2\sqrt{3}(h_1 + h_2) x_3 y_1 \\ &+ 2\sqrt{3} h_1 h_2 x_2 x_3 + 4(y_0^2 + y_1^2 + y_2^2 + y_3^2) - (x_0^2 + x_1^2 + x_2^2 + x_3^2) r_3^2 = 0. \end{aligned}$$

Finding the operation modes?

- Ex: the 3-RPS manipulator. Primary decomposition (using sofware Singular):

$$\mathcal{J} = \bigcap_{i=1}^{3} \mathcal{J}_i,$$

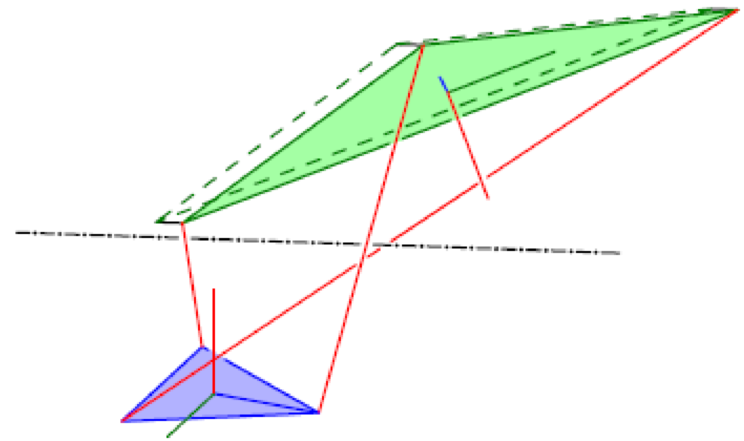
with

$$\begin{aligned}
\mathcal{J}_1 = \langle & x_0, x_1y_1 + x_2y_2 + x_3y_3, h_2x_2^2 - h_2x_3^2 - 2x_1y_2 + 2x_2y_1 + 2x_3y_0, \\
& h_2x_2x_3 + x_1y_3 + x_2y_0 - x_3y_1 \rangle \\
\mathcal{J}_2 = \langle & x_1, x_0y_0 + x_2y_2 + x_3y_3, h_2x_2^2 - h_2x_3^2 - 2x_0y_3 + 2x_2y_1 + 2x_3y_0, \\
& h_2x_2x_3 - x_0y_2 + x_2y_0 - x_3y_1 \rangle \\
\mathcal{J}_3 = \langle & x_0, x_1, x_2, x_3 \rangle .
\end{aligned}$$

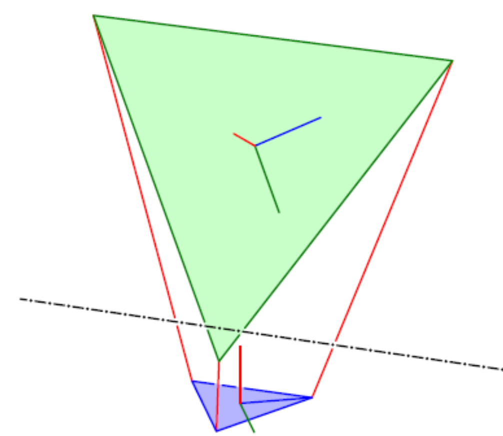
- The last one has no solutions so there are 2 operations defined by x_0=0 and x_1=0

Finding the operation modes?

- x_0=0: this is the so-called π-screw motion, i.e., a screw motion of angle 180° about any axis => the platform is tilted most often



- x_1=0: a screw about any axis lying in the horizontal plane



Schadlbauer, J., Nurahmi, L., Husty, M., Wenger, P. and Caro, S., "Operation Modes in Lower-Mobility Parallel Manipulators", Second Conference on Interdisciplinary Applications of Kinematics, Lima, Peru, September 9–11, 2013.

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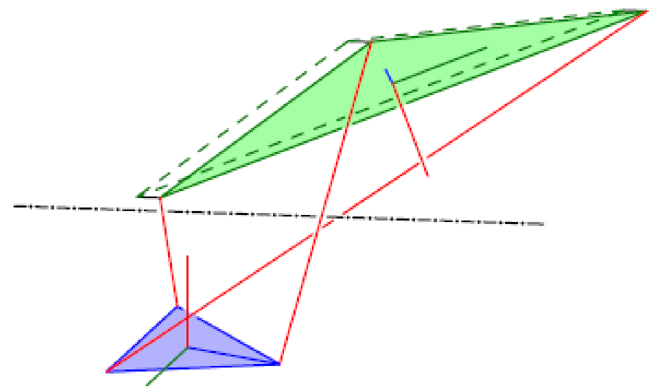
Finding the operation modes?

- For the 3-R$\underline{\text{P}}$S, the motions modes can be parametrized as shown in the frame of PHC project IRCCyN/Innsbrück Univ.
- For x_0=0:



$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a & 1-2(\cos^2 u - \sin^2 u \cos^2 v) & 2\sin u \sin v \cos u & 2\sin^2 u \sin v \cos v \\ b & 2\sin u \sin v \cos u & -1+2\cos^2 u & 2\cos u \sin u \cos v \\ c & 2\sin^2 u \sin v \cos v & 2\cos u \sin u \cos v & 2\cos^2 v \sin^2 u - 1 \end{pmatrix}$$

where:

$$a = \frac{2(-y_3 - 2\sin v\ \cos v\ \cos u + 2\sin v\ \cos v\ \cos^3 u)}{\cos u}$$

$$b = -4\cos u\ \sin u\ \cos v$$

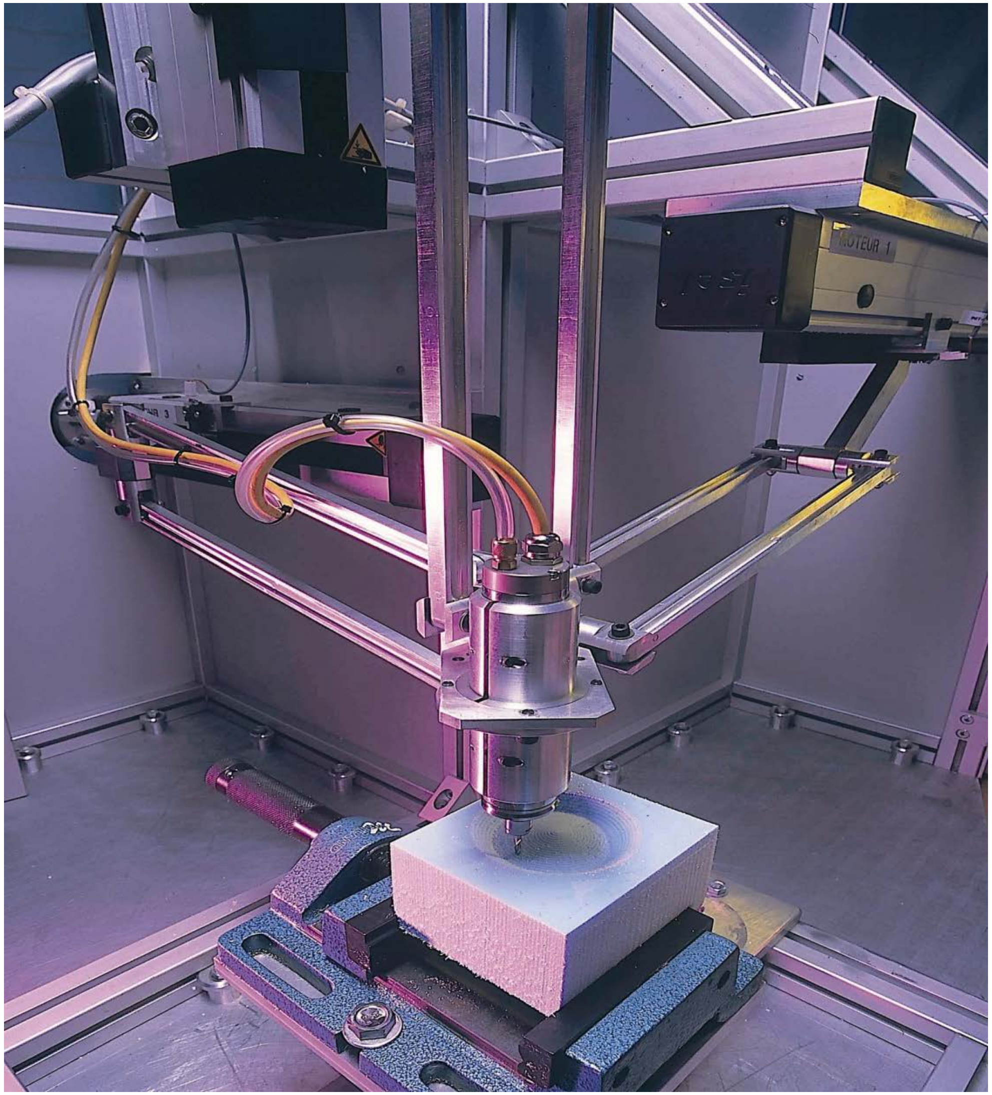
$$c = 2(\cos^2 v\ \sin^2 u - \cos^2 u)$$

Avoiding the singularities?

- At the design stage (architecture and geometry, actuation or kinematic redundancy, actuation mode, …)
- By restricting the workspace or joint space
- By choosing « good » trajectories

Avoiding the singularities?

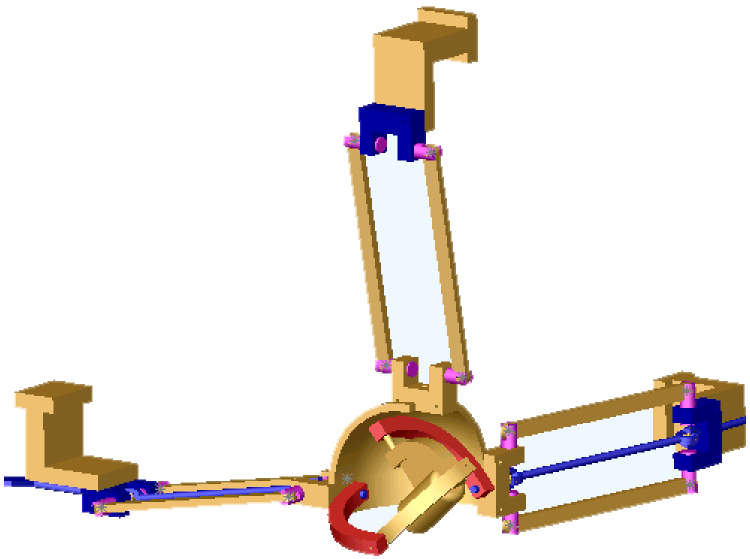
- Select kinematic and geometric parameters to eliminate any singularity
- Example: design isotropic manipulators (Orthoglide, IRCCyN) or fully-isotropic manipulators (Tripteron, Laval University)

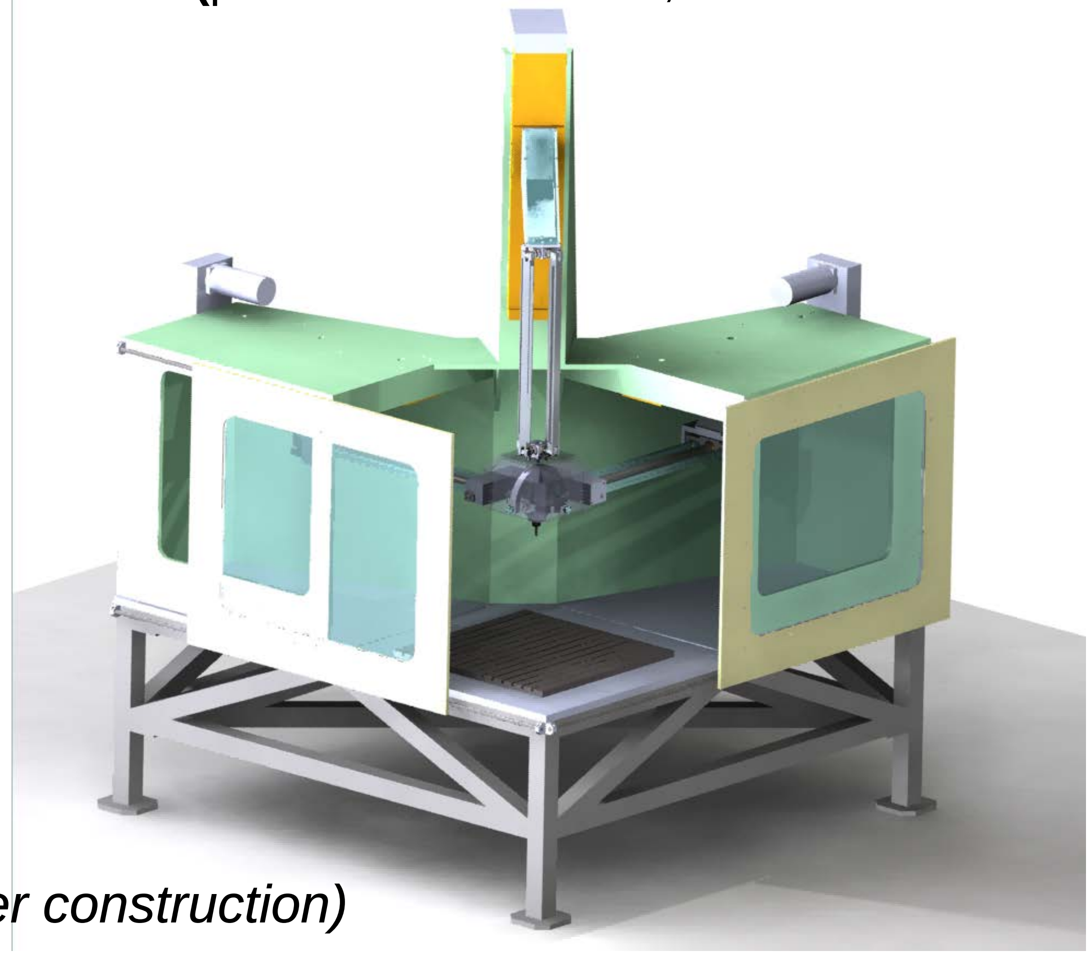


3-axis Orthoglide prototype built at IRCCyN

The Input/output Jacobian matrix is isotropic in the « orthogonal » configuration

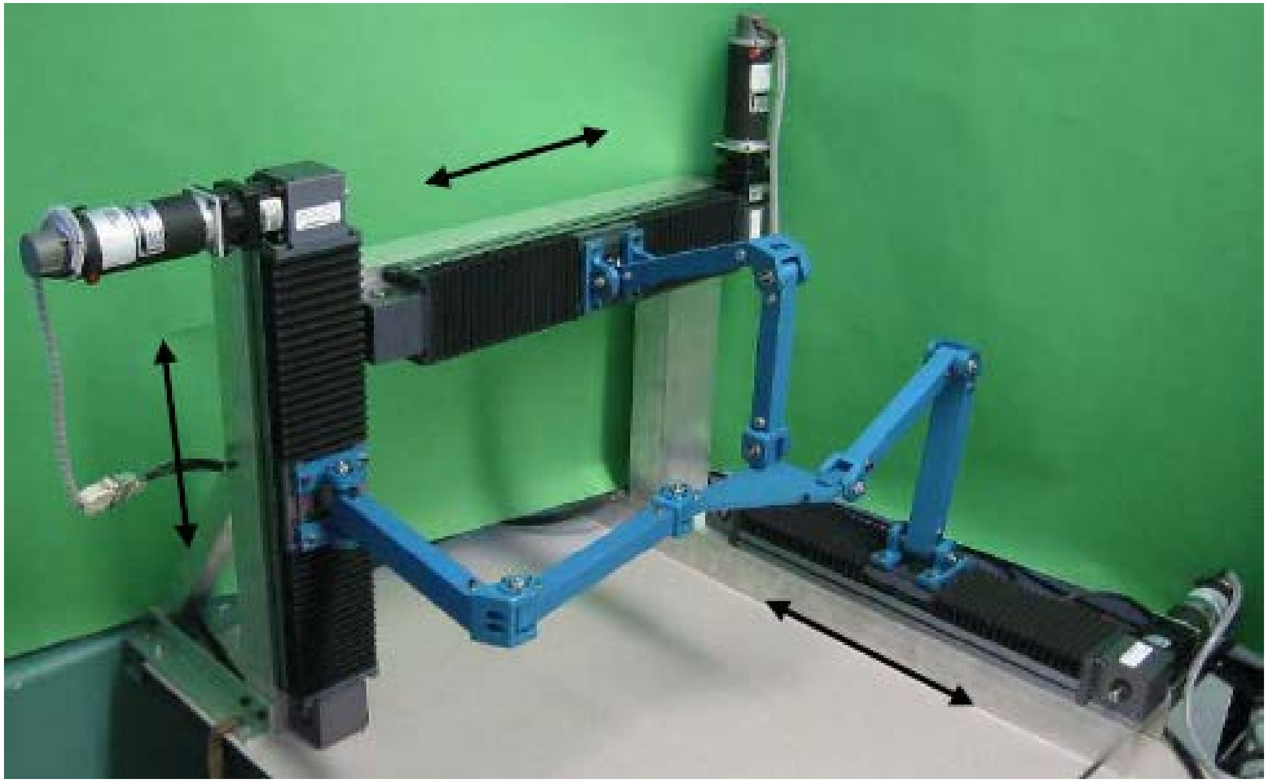
The 5-axis Orthoglide machine (patents : EP1597017, CA2515024 US20070062321)





Full-size prototype under construction)
(workspace 50x50x50)

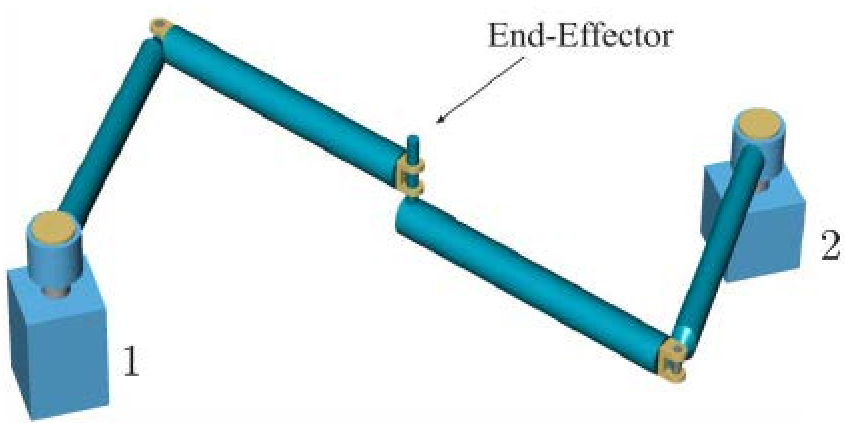
The Tripteron (Gosselin et al, Laval University, also studied by Tsai and Gogu)

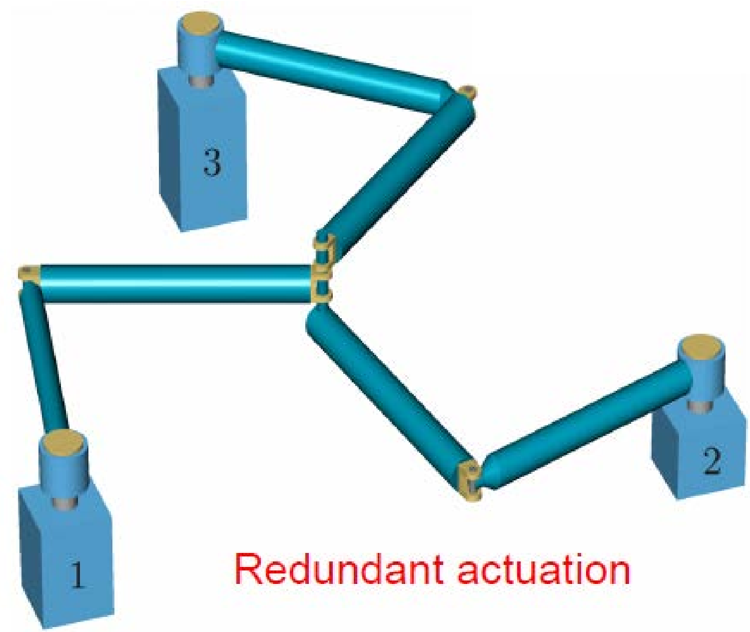


The input/output Jacobian matrix is isotropic everywhere (fully-isotropic design)

Avoiding the singularities?

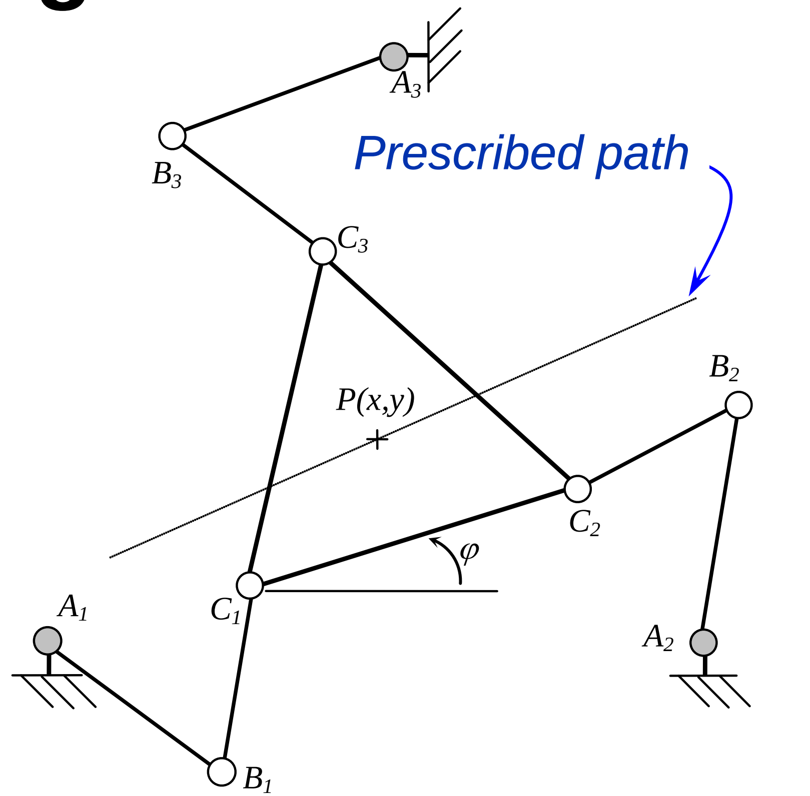
- Using actuation redundancy (see for instance work done at LIRMM, Seoul, Tokyo, …))
- Difficulty: control of the internal stresses





Avoiding the singularities?

- Using kinematic redundancy
- The moving platform is controlled with n DOF while only $m<n$ need be controlled (uncompletely specified task)
- Example: a 3-RPR manipulator used for positionning tasks in the plane (orientation not prescribed)



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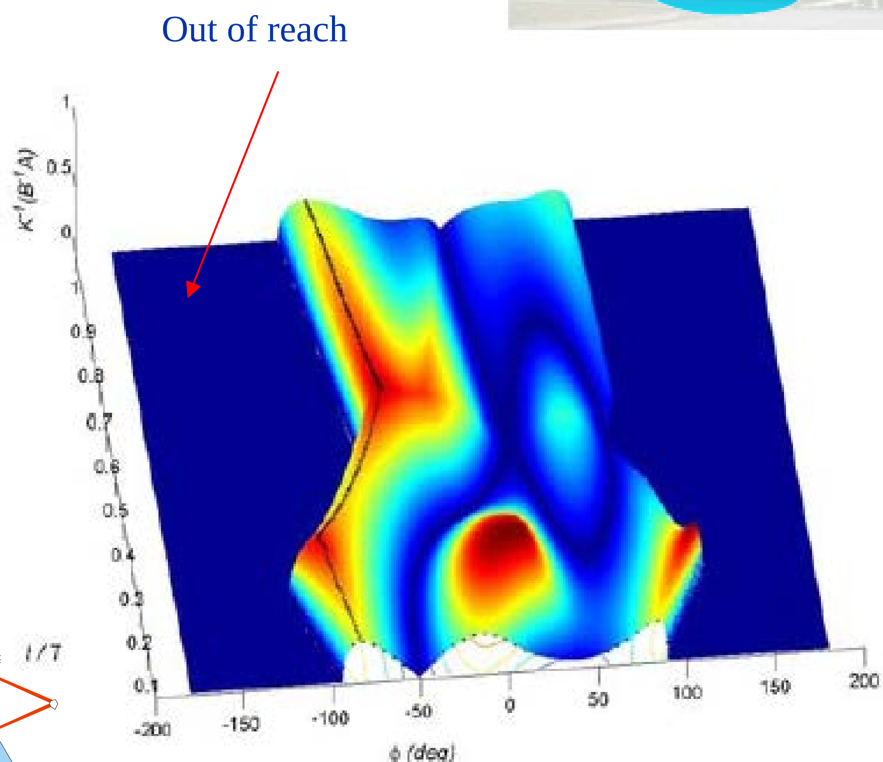
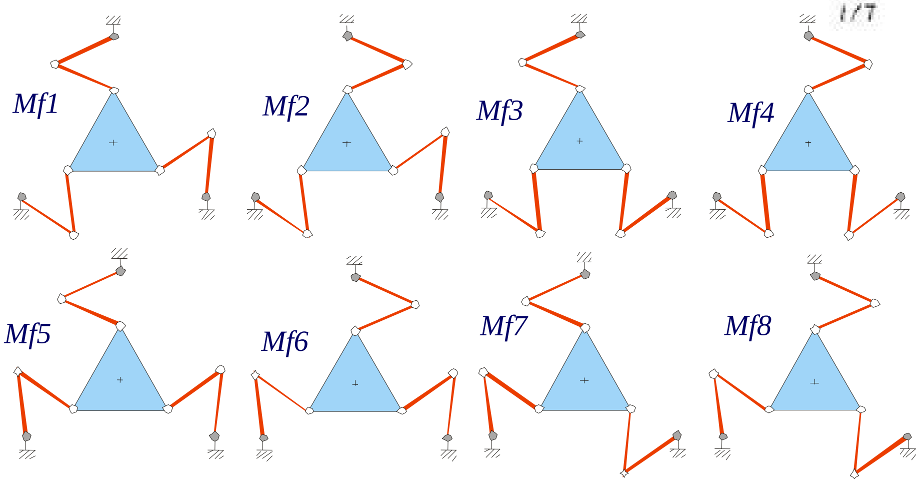
Feasibility map (FM)

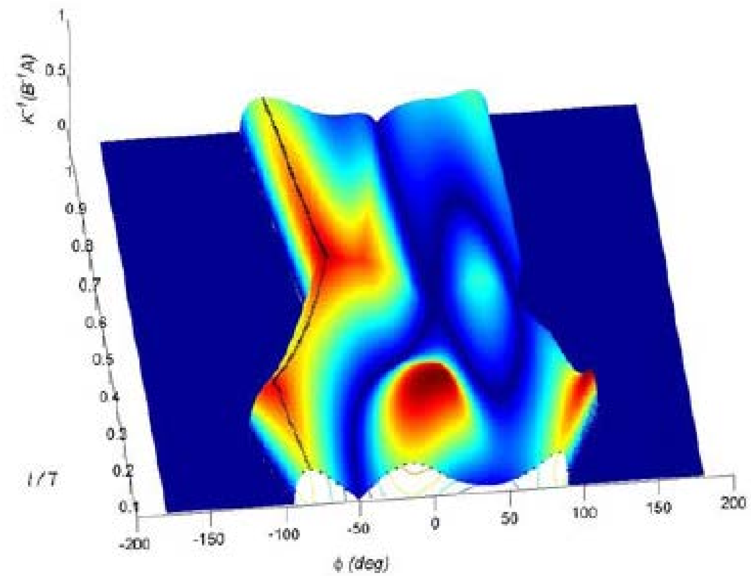
- Surface plot of $\kappa^{-1}(A_n)$ versus nondimensional time t/T and the platform orientation φ
- Provides global information
- There exists one FM per working mode





Optimal kinematic inversion along the path

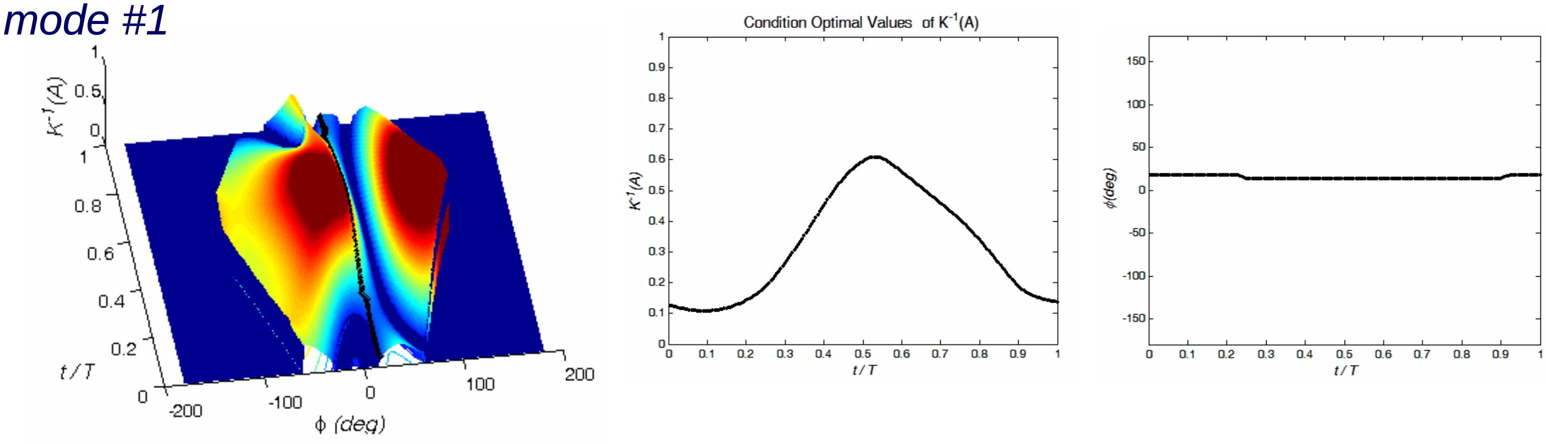
- Problem similar to planning an optimal trajectory for a vehicle moving on a complex terrain
- Vehicle should avoid hills and forbidden areas and is subject to a minimum turning radius
- Change $\kappa^{-1}(A_n)$ to $1-\kappa^{-1}(A_n)$: valleys are regions where $\kappa^{-1}(A_n)$ is high.
- An existing algorithm has been adapted (O. Alba Gomez, Ph.D, 2007)



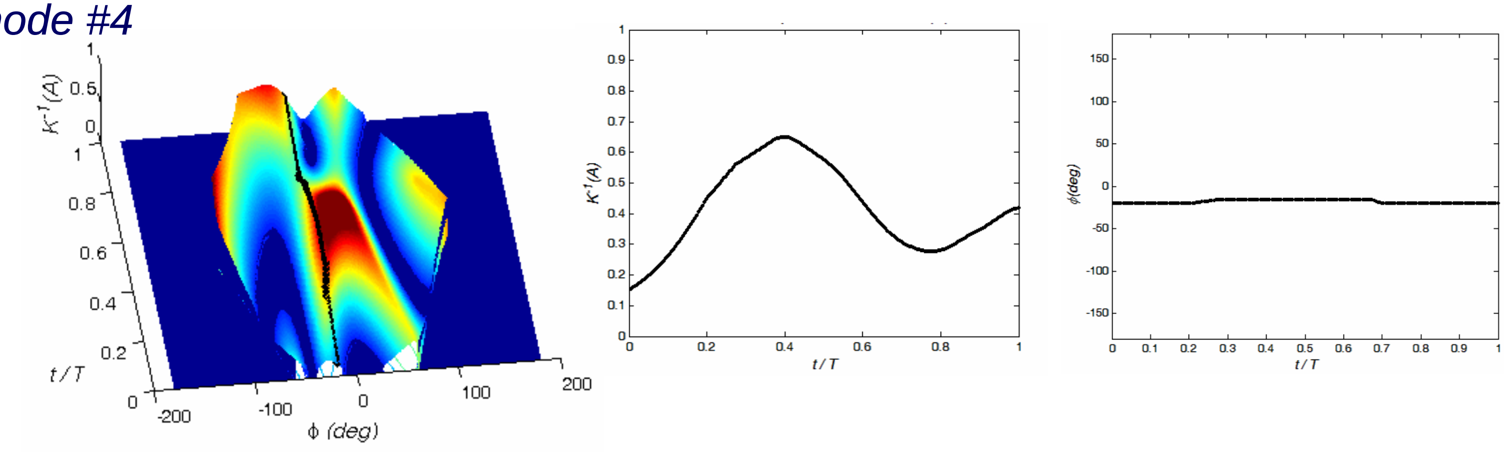
O. Alba-Gomez, A. Pamanes et P. Wenger : "Trajectory planning of a redundant parallel manipulator changing of working mode". 12th World Congress in Mechanism and Machine Science, Juin, 2007, Besançon, IFToMM

Generate an optimal trajectory in each WM and choose the best one

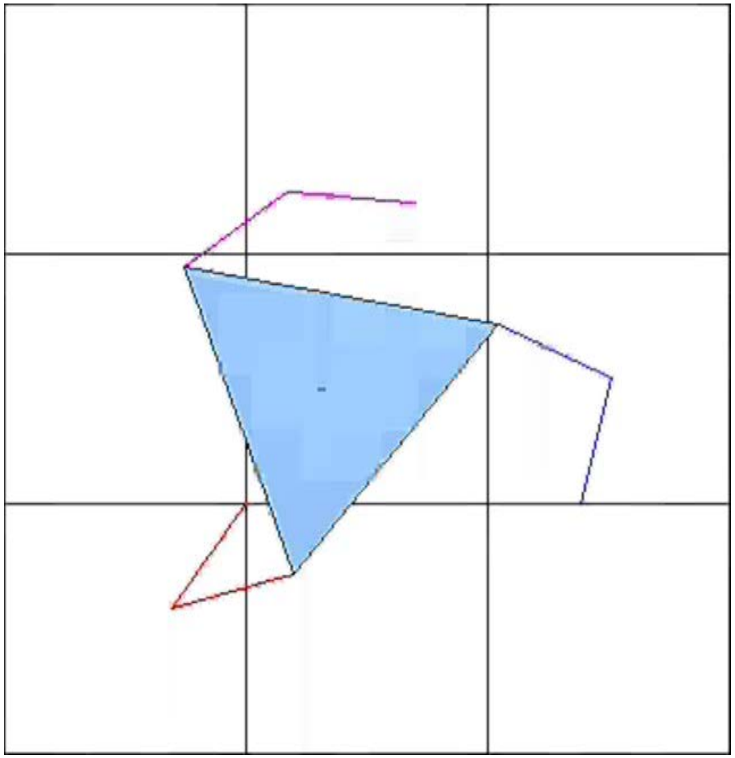
Working mode #1



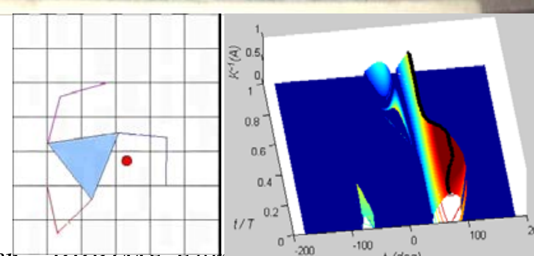
Working mode #4



Simulation and prototype trajectory

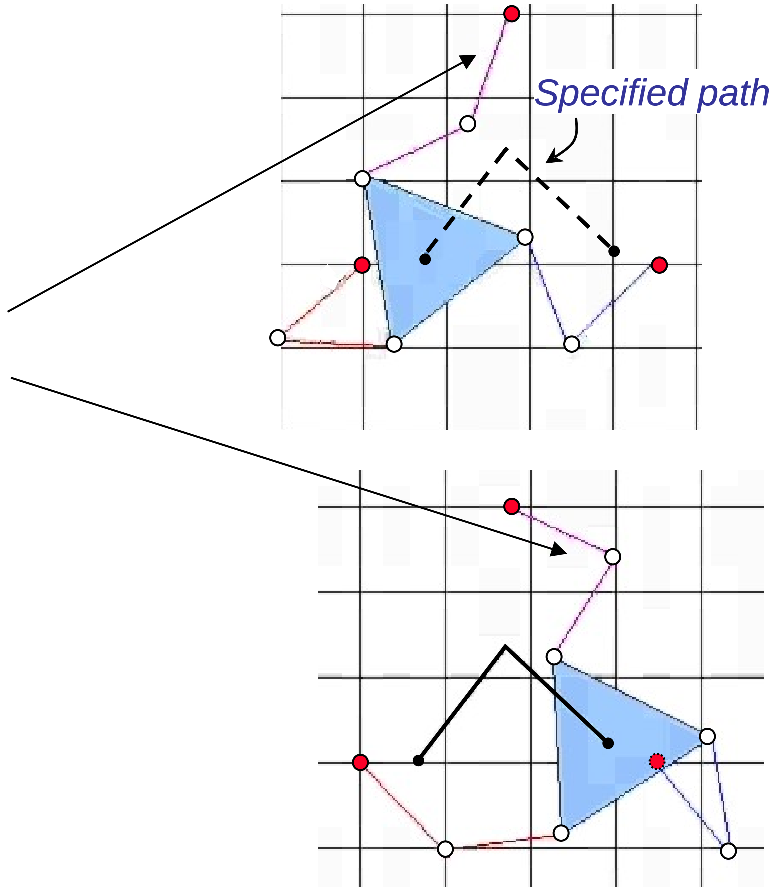


Note: joint limits and obstacles can be considered



A change of working-mode may help :

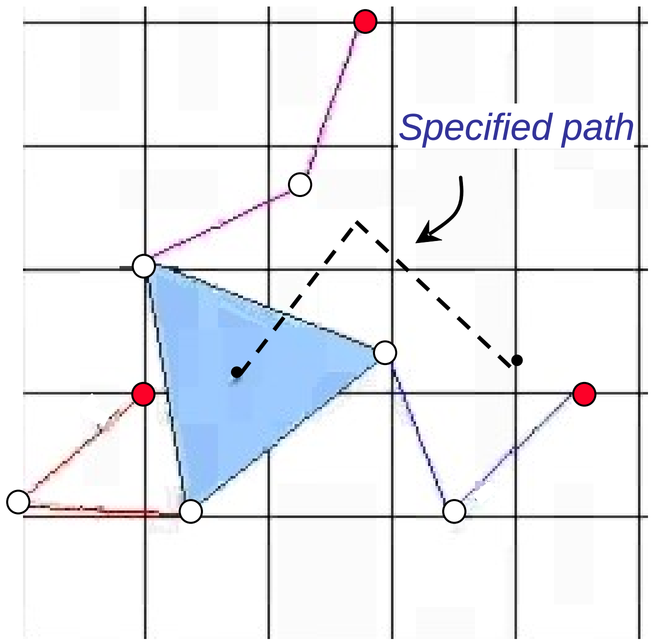
Leg posture has changed



Initial pose

Final pose

Example: path cannot be followed in one single WM (a singularity is always encountered)



Find a trajectory in two FMs associated with two distinct WMs

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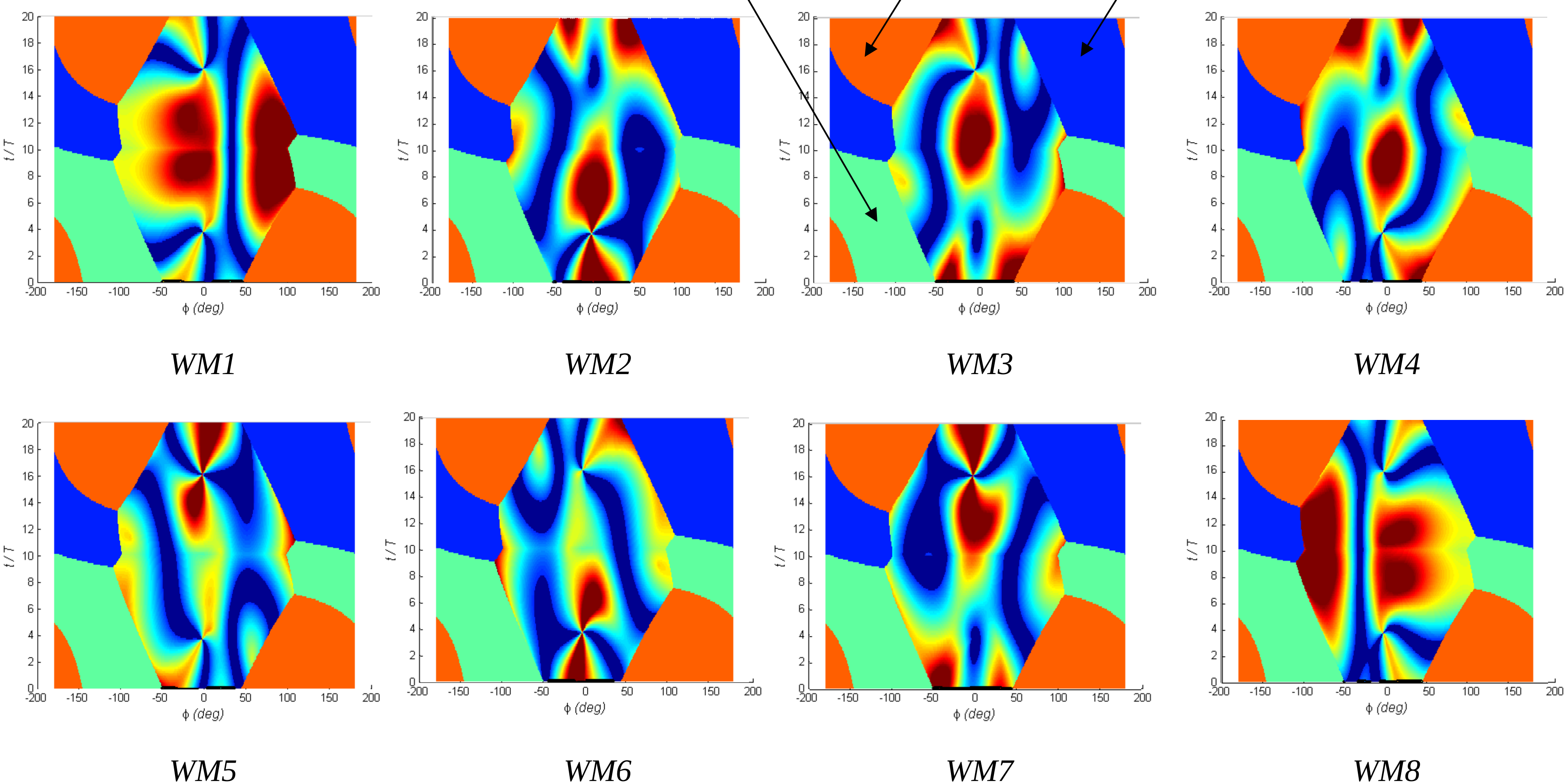
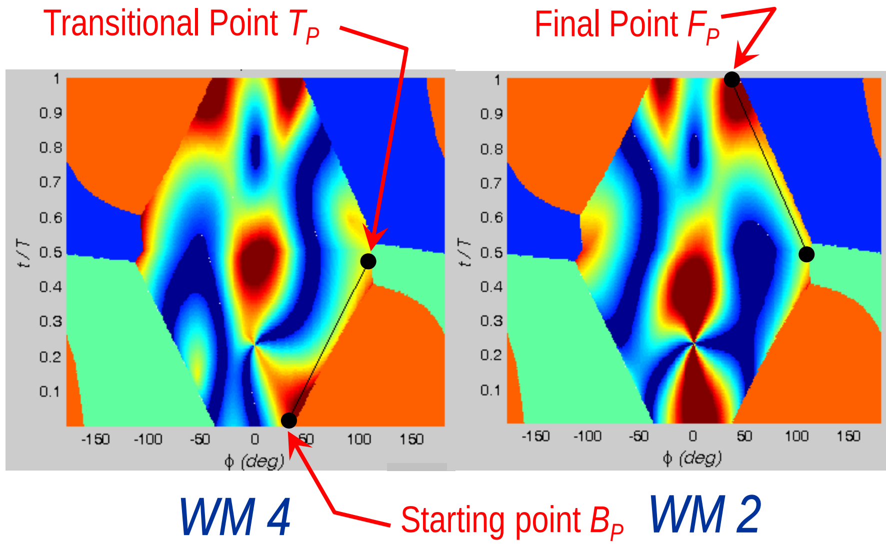
dark red : $k^{-1}(A)$ is high

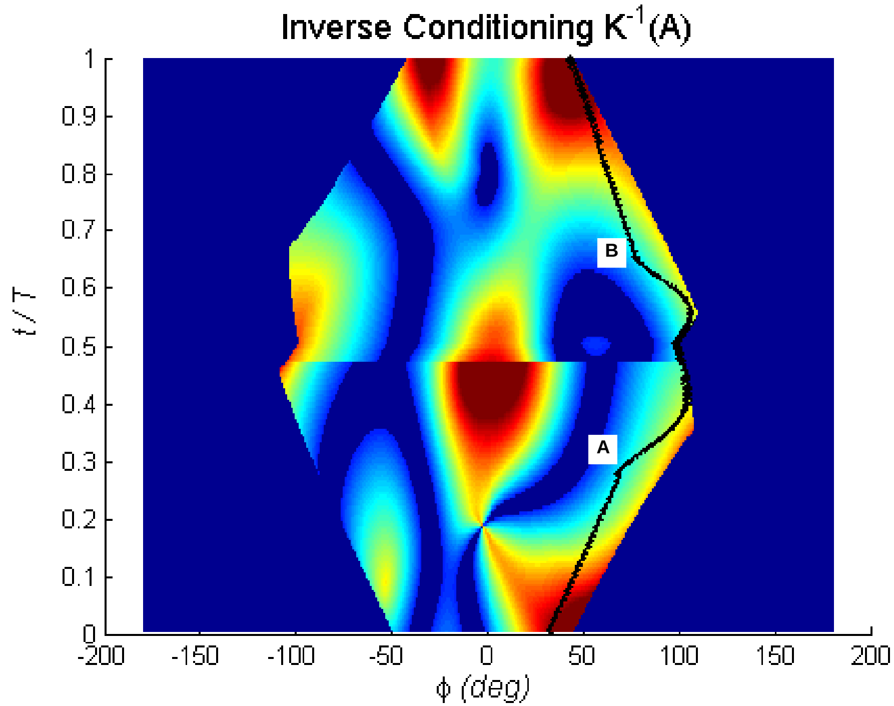
dark blue : $k^{-1}(A)$ is low

Out of reach (leg 2) Out of reach (leg 3) Out of reach (leg 1)



- *WM2* and *WM4* can be connected. Motion starts with poses in *WM4* and stops with poses in *WM2*
- Specify starting point *Bp* , transitional point *Tp* and final point *Fp* of the trajectory into the corresponding maps.
- These points are joined by right lines in such a way that values of $\kappa(A)^{-1}$ are as high as possible.
- Transitional point *Tp* must be on the border associated with a serial singularity of Second Leg in order that *WM4* and *WM2* can be exchanged





Simulation

- A cycloidal law is applied to smooth the trajectory at the transitional point

Avoiding the singularities?

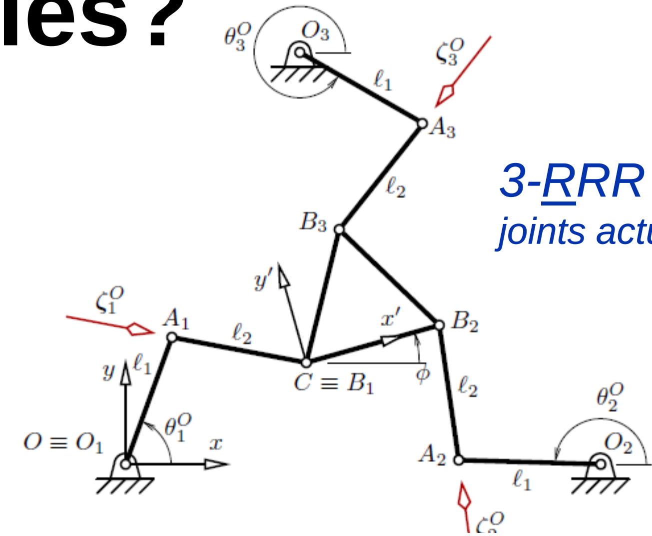
- Using actuation mode

- Singularities depend on the choice of the actuated joints

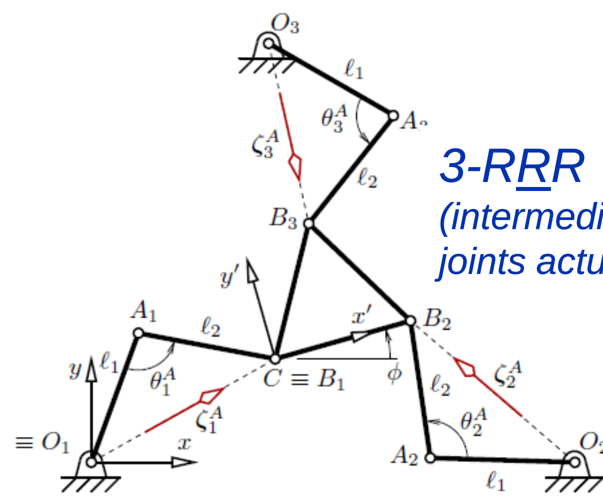
- The idea is to select an appropriate set of actuated joints according to the prescribed path

- The goal is to keep the path as far as possible from any singularity for good kinetostatic performances

- Play with the actuation of the first and second R joint in each RRR chain



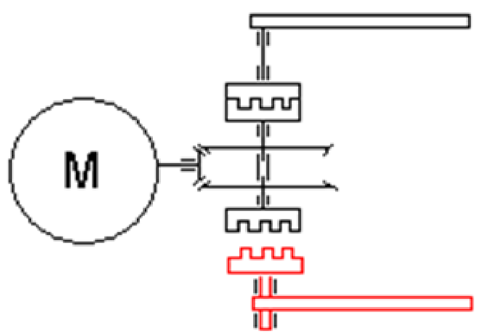
3-RRR (base joints actuated)

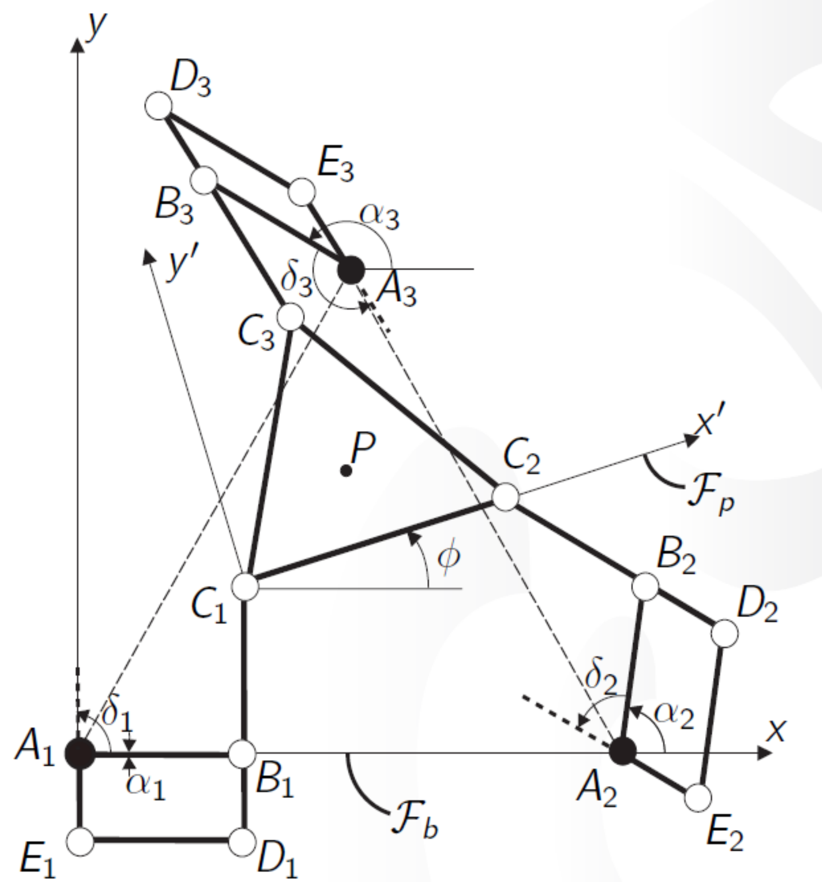


3-RRR (intermediate joints actuated)

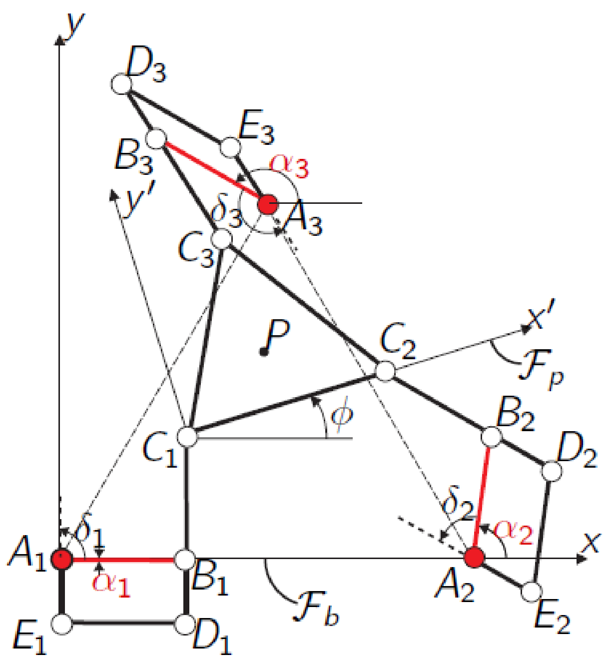
Manipulator with variable actuation: concept design

- Concept design: proximal link is replaced by a parallelogram, in which the first two links can be driven independently
- The driven links are selected by means of two clutches mounted on each base joint
- Thus only three actuators are needed





Manipulator with variable actuation: 8 actuation modes



TAB.: The eight actuating modes of the 3-RRR VAM

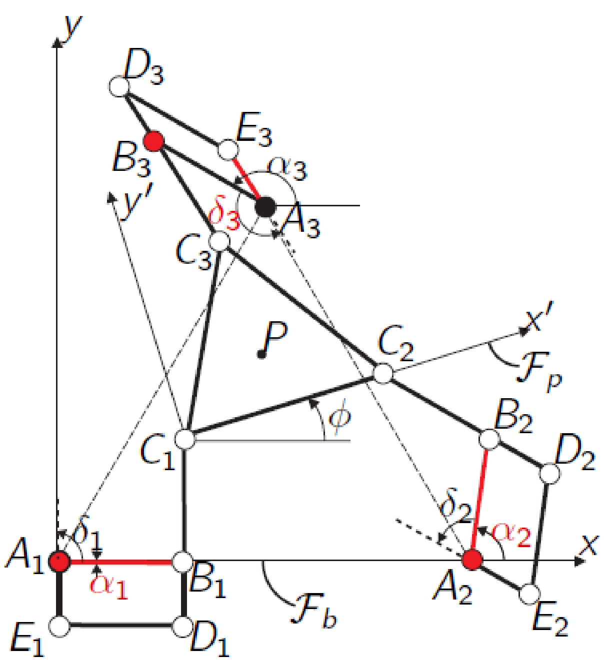
	driven links	active angles
1	A_1B_1, A_2B_2, A_3B_3	α_1, α_2, α_3
2	A_1B_1, A_2B_2, A_3E_3	α_1, α_2, δ_3
3	A_1B_1, A_2E_2, A_3B_3	α_1, δ_2, α_3
4	A_1E_1, A_2B_2, A_3B_3	δ_1, α_2, α_3
5	A_1B_1, A_2E_2, A_3E_3	α_1, δ_2, δ_3
6	A_1E_1, A_2E_2, A_3B_3	δ_1, δ_2, α_3
7	A_1E_1, A_2B_2, A_3E_3	δ_1, α_2, δ_3
8	A_1E_1, A_2E_2, A_3E_3	δ_1, δ_2, δ_3

⇕

3 – RRR manipulator

Rakotomanga, N., Chablat, D. and Caro, S., "Kinetostatic Performance of a Planar Parallel Mechanism with Variable Actuation", Advances in Robot Kinematics, Batz-sur-Mer, France, June 22-26, 2008, pp. 311–320. hal-00322760

Manipulator with variable actuation: 8 actuation modes



TAB.: The eight actuating modes of the 3-RRR VAM

	driven links	active angles
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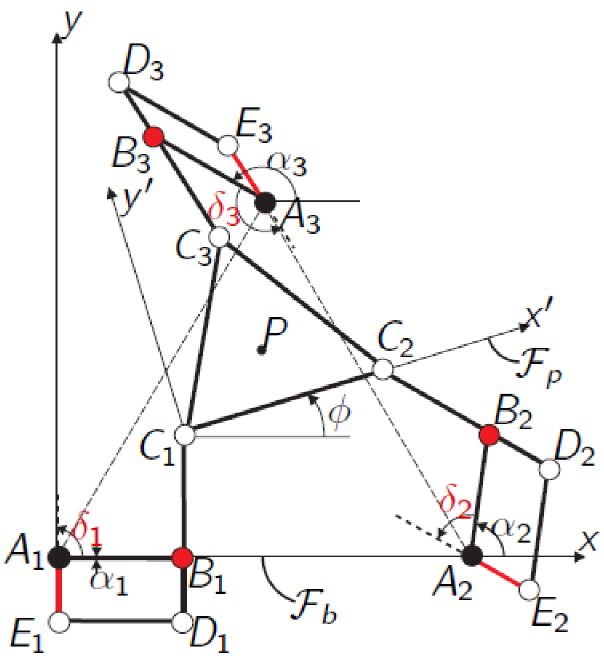
2	A_1B_1, A_2B_2, A_3E_3	α_1, α_2, δ_3
3	A_1B_1, A_2E_2, A_3B_3	α_1, δ_2, α_3
4	A_1E_1, A_2B_2, A_3B_3	δ_1, α_2, α_3
5	A_1B_1, A_2E_2, A_3E_3	α_1, δ_2, δ_3
6	A_1E_1, A_2E_2, A_3B_3	δ_1, δ_2, α_3
7	A_1E_1, A_2B_2, A_3E_3	δ_1, α_2, δ_3
8	A_1E_1, A_2E_2, A_3E_3	δ_1, δ_2, δ_3

Manipulator with variable actuation: 8 actuation modes



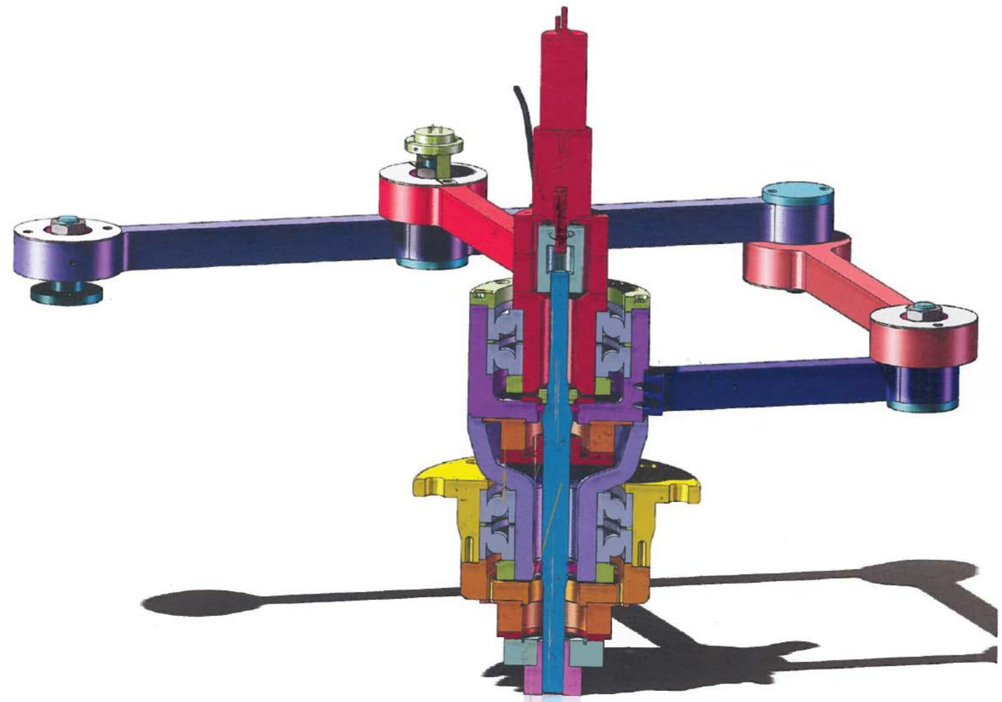
TAB.: The eight actuating modes of the 3-RRR VAM

	driven links	active angles
1	A_1B_1, A_2B_2, A_3B_3	α_1, α_2, α_3
2	A_1B_1, A_2B_2, A_3E_3	α_1, α_2, δ_3
3	A_1B_1, A_2E_2, A_3B_3	α_1, δ_2, α_3
4	A_1E_1, A_2B_2, A_3B_3	δ_1, α_2, α_3
5	A_1B_1, A_2E_2, A_3E_3	α_1, δ_2, δ_3
6	A_1E_1, A_2E_2, A_3B_3	δ_1, δ_2, α_3
7	A_1E_1, A_2B_2, A_3E_3	δ_1, α_2, δ_3
8	A_1E_1, A_2E_2, A_3E_3	δ_1, δ_2, δ_3

$\Updownarrow$

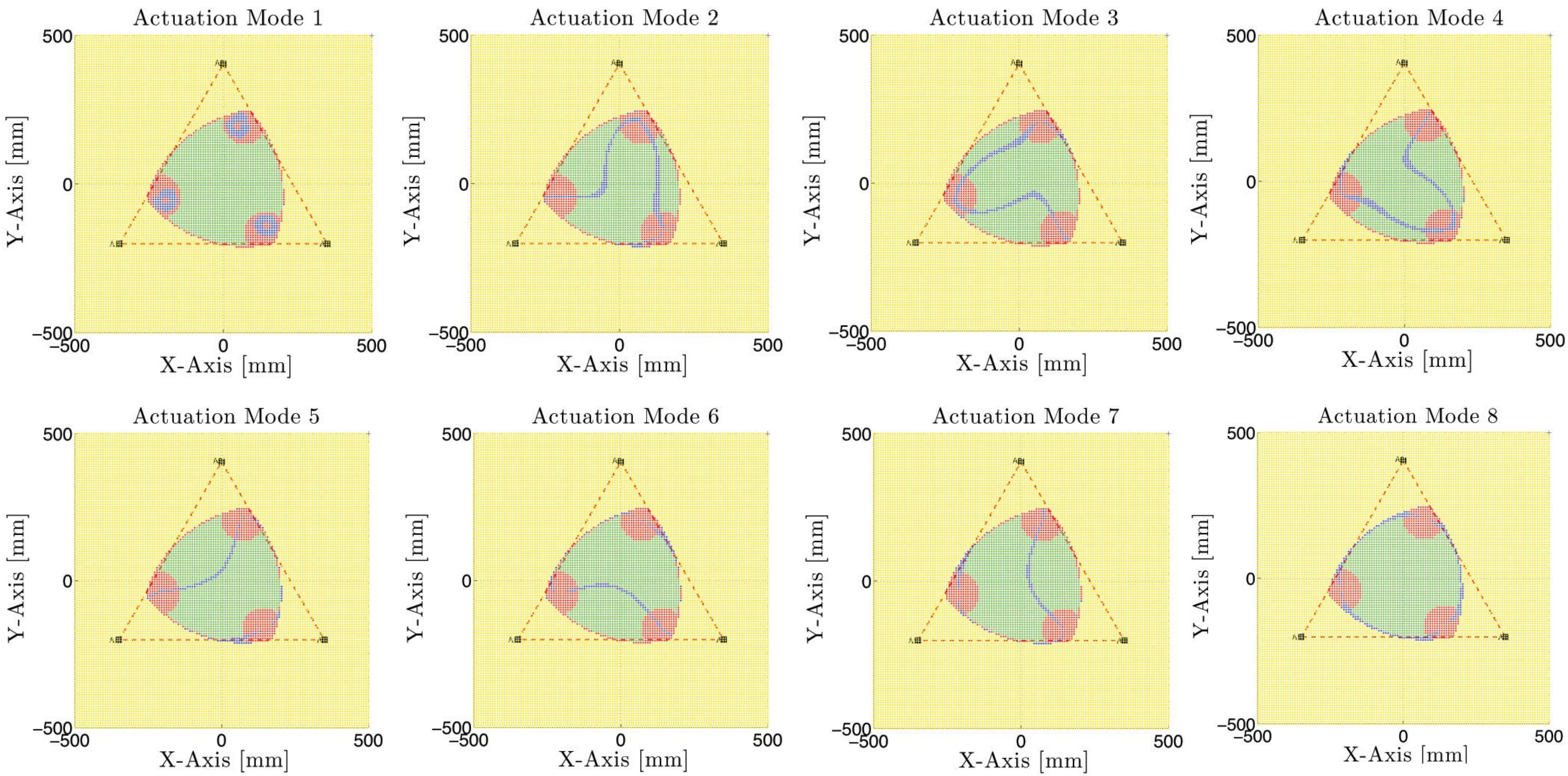
3 – R$\underline{\text{R}}$R manipulator

NaVARo : Nantes Variable Actuation Robot



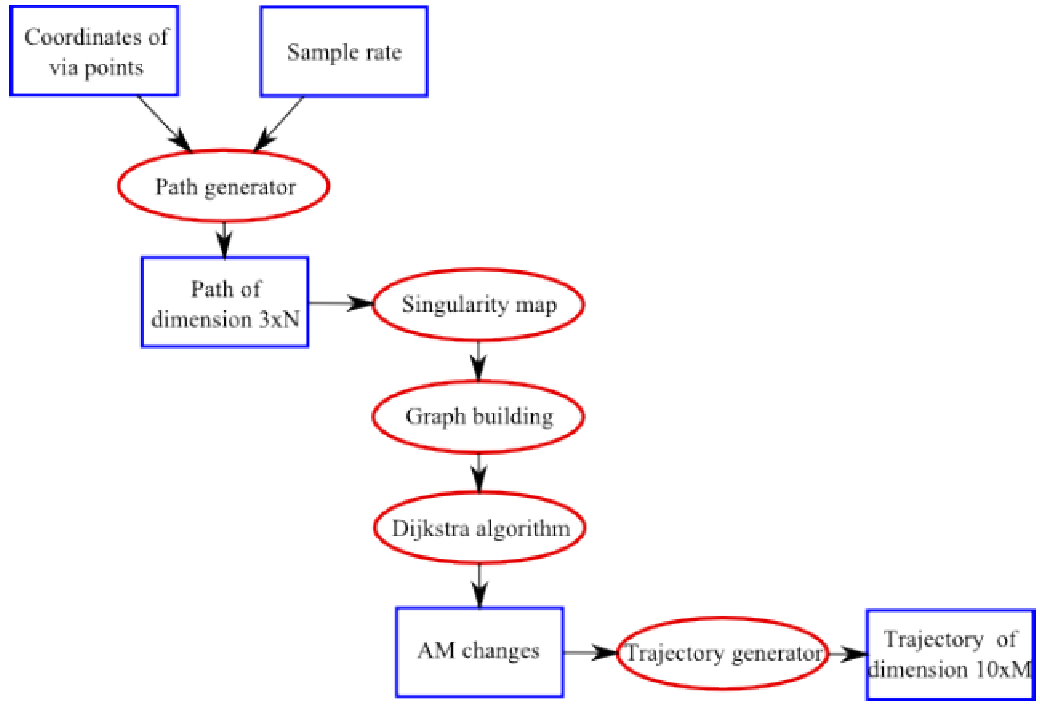
Caro, S., Chablat, D., Wenger, P., and Kong, X., 2014, “Kinematic and Dynamic Modeling of a Parallel Manipulator with Eight Actuation Modes”, New Trends in Medical and Service Robots, Springer, Book ISBN : 978-3-319-05430-8 pp. 315–329.

Workspace and singularity analysis (ϕ=20°)



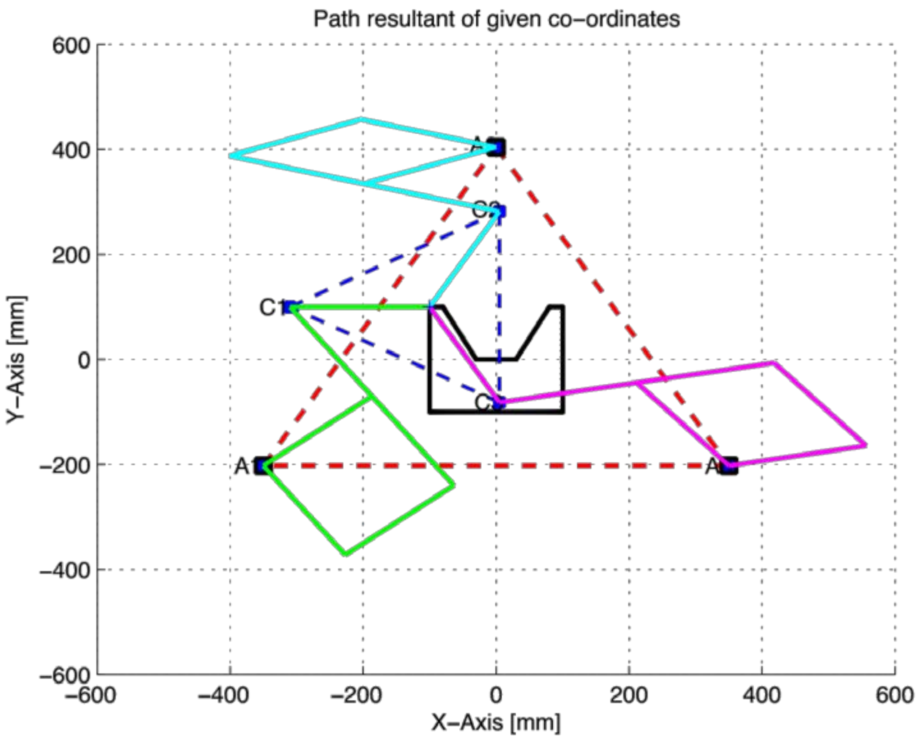
Actuation mode selection

- Given a path composed of N points connected with linear segments, select an optimal actuation mode in each segment
- Optimal: constraint on a kinetostatic performance index (« distance » to singularities)
- Minimize the number of actuation mode changes
- Additional criteria: minimize travelling time, energy consumption, …



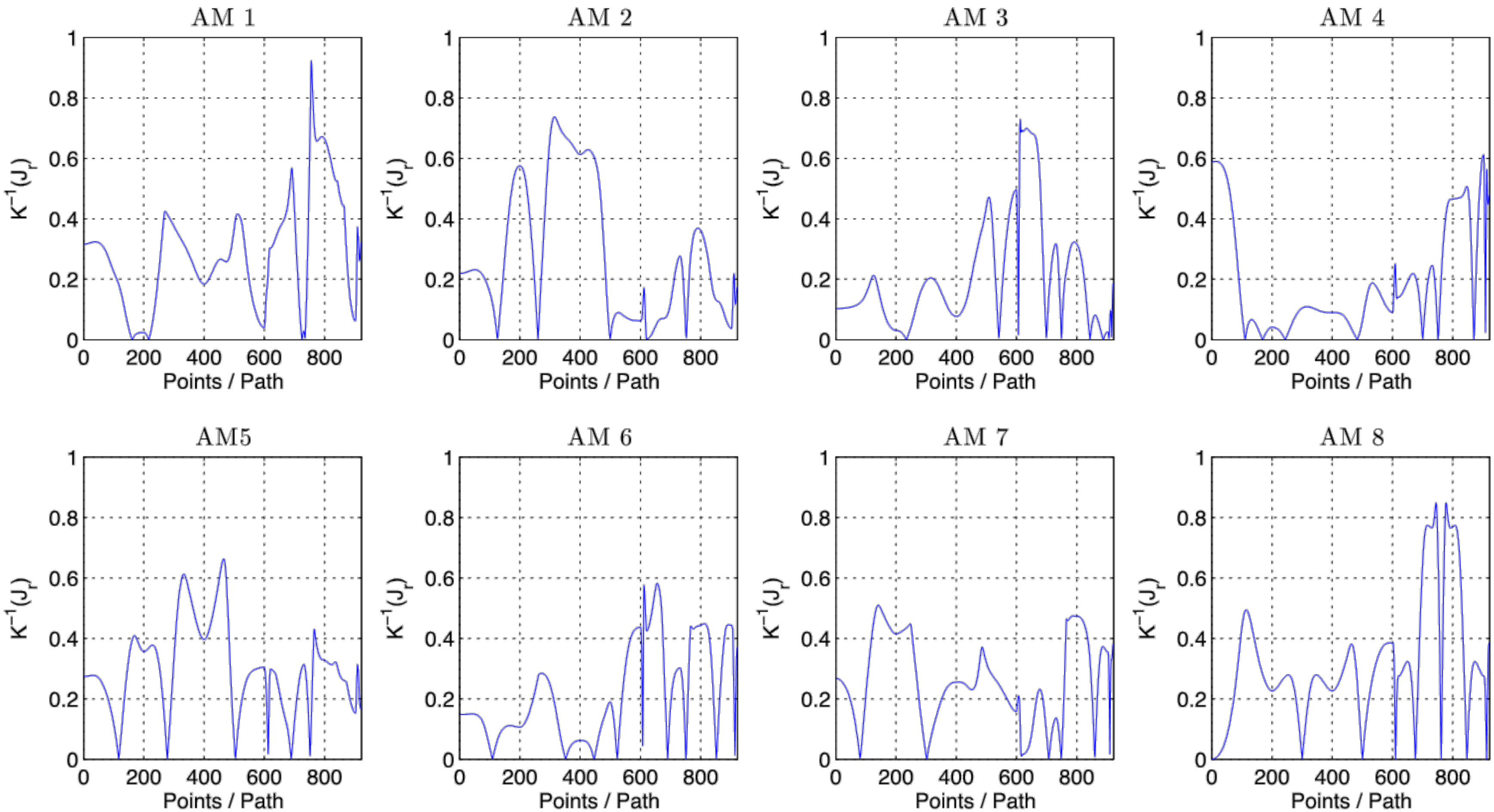
Caro, S., Chablat, D. and Hu, Y., "Algorithm for the Actuation Mode Selection of the Parallel Manipulator NaVARo", Proceedings of the ASME 2014 International De-sign Engineering Technical Conferences & Computers and Information in Engineering
Conference IDETC/CIE 2014, Buffalo, NY, USA, August 17–

Test trajectory



X-min	Y-min	Φ-min	X-max	Y-max	Φ-max
-100	100	0	-100	-100	π/6
-100	-100	π/6	100	-100	-π/6
100	-100	-π/6	100	100	π/6
100	100	π/6	80	100	-π/6
80	100	-π/6	30	0	π/6
30	0	π/6	-30	0	-π/6
-30	0	-π/6	-80	100	π/6
-80	100	π/6	-100	100	-π/6

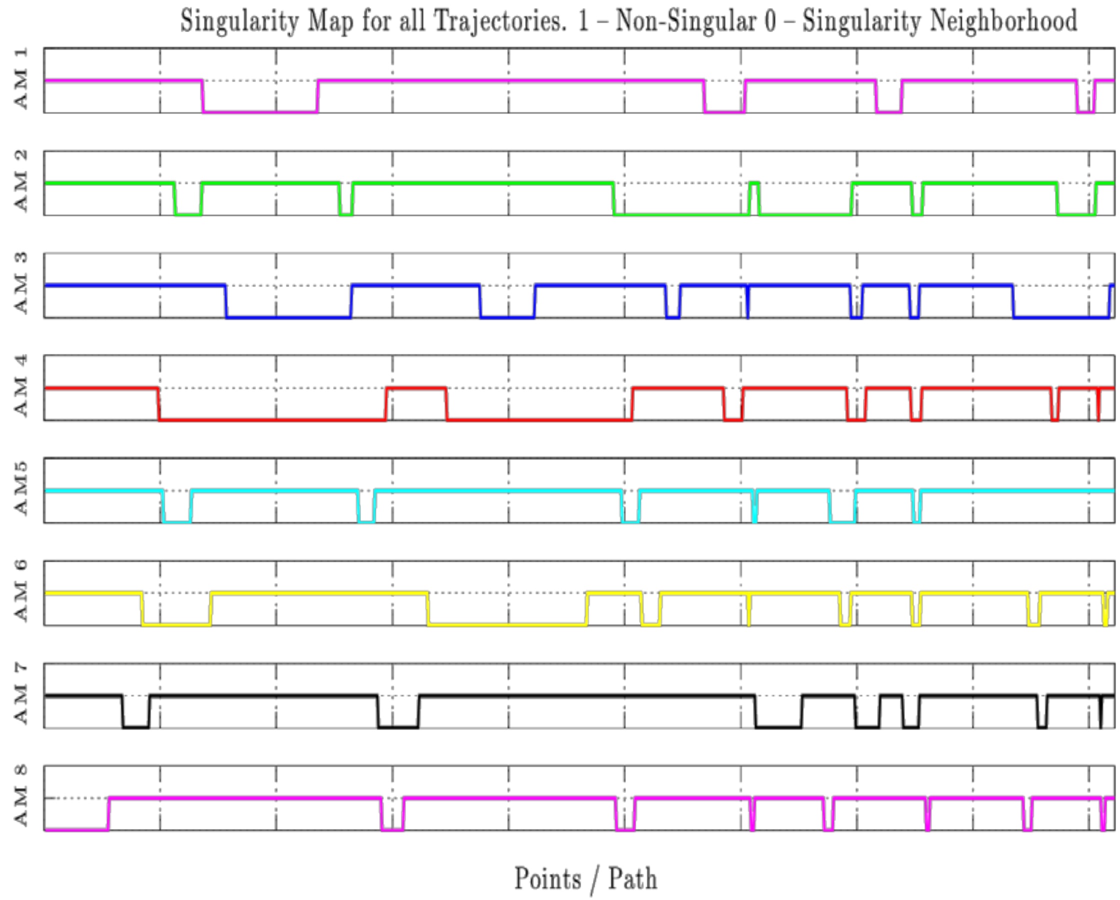
Condition number along the test trajectory for each AM



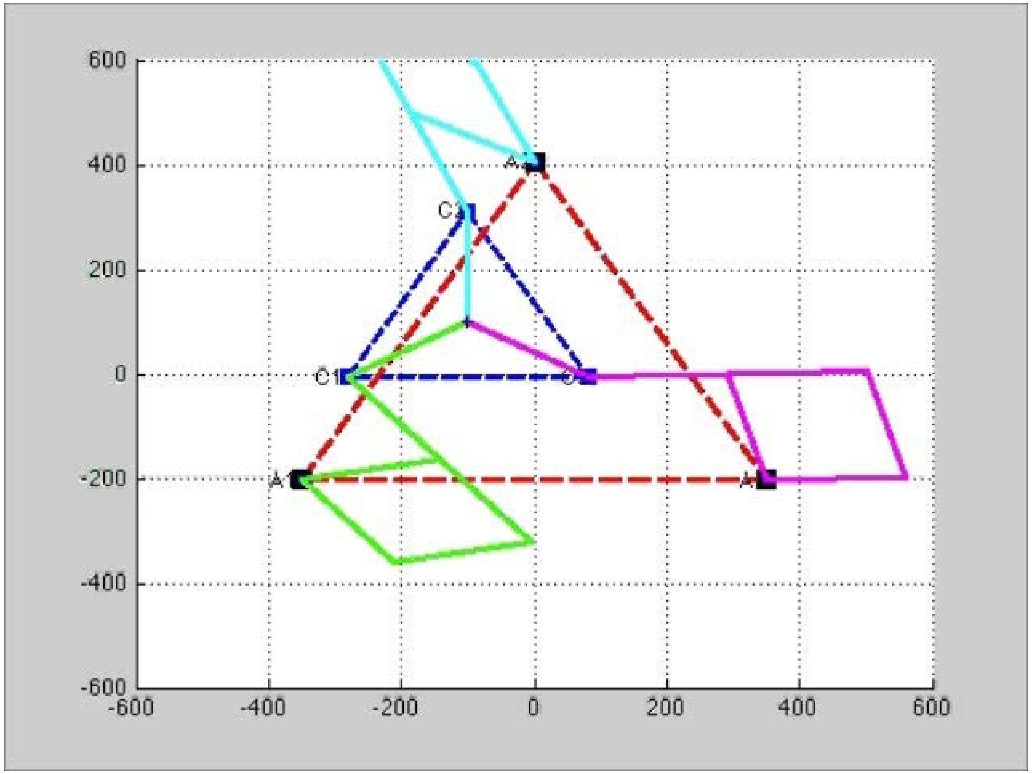
Singularity map and best sequence found

- Starting AM cannot be #8
- Number of actuation modes: 6
- Best sequence:

3 - 6 - 7 - 1 - 8 - 5



Simulation



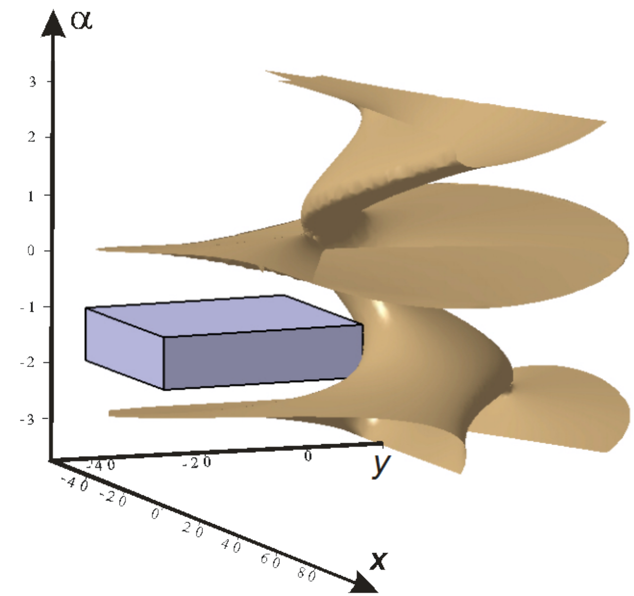
X-min	Y-min	Φ-min	X-max	Y-max	Φ-max
-100	**100**	**0**	**-100**	**-100**	**π/6**
-100	**-100**	**π/6**	**100**	**-100**	**-π/6**
100	**-100**	**-π/6**	**100**	**100**	**π/6**
100	**100**	**π/6**	**80**	**100**	**-π/6**
80	**100**	**-π/6**	**30**	**0**	**π/6**
30	**0**	**π/6**	**-30**	**0**	**-π/6**
-30	**0**	**-π/6**	**-80**	**100**	**π/6**
-80	**100**	**π/6**	**-100**	**100**	**-π/6**

First experiments

Square path with constant orientation

Avoiding the singularities?

- Using a regular dextrous workspace
- Restrict motion in a region of regular shape where the kinetostatic performances are satisfactory
- Example: for a given 3-R<u>P</u>R manipulator, find the maximal parallelepiped with a given $\Delta\alpha_0$ (Zein, Ph.D, 2007)
- Can be also done in the joint space



Avoiding the singularities?

- Singularities can be crossed?
- Under some conditions (dynamics, trajectory), see recent work of Pagis (IRCCyN/Inst. Pascal, Ph.D defense in next January)
- See also the work of Nenchev (Robotica, 1998)
- Using an external action (gravity, spring, …)

Conclusions

- Singularities cannot be ignored when designing complex mechanisms
- Many types of singularities exist
- Needs efficient mathematical tools
- Difficult to detect them in a systematic way
- Often difficult to get a geometric interpretation useful for the designer
- THANK YOU FOR YOUR ATTENTION!

Questions, discussion…