## PPRIME FORUM

«Mechanical Design and Mechatronics of robotics systems»

```
Futuroscope, November, 2014
Lecture hall, SP2MI building
```

rue Gustave Eiffel Futuroscope Chasseneuil, France
*_**

The FORUM organized by the Pprime Institute offers the opportunity to PhD students and researchers from the robotics community to meet experts in order to exchange with them about most recent scientific results. This event provides to participants a space of reflection and privileged exchange.

The FORUM is dedicated to Mechanical Design and Mechatronics of robotics systems. Two themes, in the field of interest of the robotics team of Pprime institute, are considered during this FORUM.

- The design of mechanical hands for dexterous manipulation,
- The design of complex poly-articulated mechanisms (parallel mechanisms).

The forum is organized over three days with a program focused on presentations and panel discussions. In this context, we will have four guest speakers:

- Yukio Takeda, Professor, Tokyo Institute of Technology, Japan
- Philippe Wenger, Directeur de recherches au CNRS, IRCCyNNantes, France
- Markus Grebenstein, doctor, DLR German Aerospace Center, Munich, Germany
- Chin-Hsing Kuo, Professor, National Taiwan University of Science and Technology, Taiwan


## PPRIME Forum

## «Mechanical Design and Mechatronics of robotics systems»

Thursday, November 6: "Kinematic optimization of complex poly-articulated systems"

## Morning: 10h-12h

- Yukio Takeda, Professor Tokyo Institute of Technology - Japon
- Title : <Kinematic Design of Compensatable Parallel Manipulators »
- Panel discussion


## Afternoon: 14h-17h

- Philippe Wenger, Directeur de recherches au CNRS IRCCyN Nantes - France.
- Title : «Coping with singularitronwemd design of parallel-manipulators»
- Panel discussion
- Yukio Takeda, Professor Tokyo Institute of Technology - Japon
- Title : « Kinematic and Dynamic Analysis and Design of 3-RPSR Parallel Mechanism for Pipe-Bender "
- Panel Discussion

Friday, Novemher 19: «Design of medical robots / Design of mechanical hands»

## Morning: 10h-12h

Chin-Hsing Kuo, Professor National Taiwan University of Science and Technology Taiwan

- Title : «Applications of Mechanism Design Theories for Surgical Robotics» - Panel Discussion


## Afternoon: 14h-17h

Sebastian Wolf, Doctor, DLR German Aerospace Center, Munich, Germany

- Title : « Design of the DLR-Hand Arm System - Focus on Variable Impedance Actuation (VIA)"
- Panel Discussion


## Contents

- Introduction of our Lab. slide
- Introduction of previous researches on parallel mechanisms in our lab. slide
- Main topic: kinematic design of compensatable parallel manipulators


## What is Tokyo Tech?

- 3 schools for undergraduates: Science, Engineering, Bioscience and Biotechnology
- 6 graduate schools: Science and Engineering, Bioscience and Biotechnology, Interdisciplinary Science and Engineering, Information Science and Engineering, Decision Science and Technology, Innovation Management
- Number of students : 4800 in undergraduate schools(860 in science, 3300 in engineering, 640 in bioscience and biotechnology, 3600 in master course of graduate schools, 1550 in doctoral course of graduate schools, 1320 from foreign countries
- Number of teaching staffs: 1150, number of administration staffs: 1700


## Where is Tokyo Tech?



## Where is Tokyo Tech?



## Views of Tokyo Tech



Ginko street (November)


Library

## Information of our Lab.

- Department: Mechanical Sciences and Engineering
- Prof. Yukio TAKEDA, Dr. Eng.
- Assist. Prof. Daisuke MATSUURA, Dr. Eng.
- Assist. Prof. Shinji TANAKA, Dr. Eng.
- Graduate Students

Master course: 4(first year)+4(second year)

- Undergraduate Students: 4
- Exchange/Research Students: 4 (Netherland, Sweden, China)
- Kinematics (Analysis and Synthesis)
- Mechanisms (Parallel Mechanism, etc.)
- Machine Elements (Joint, Brake, etc.)
- Welfare Machines (Walking Assist, Rehabilitation, etc.)
- Machines (Machine Tools, Pipe Bender)
- Positioning, Measurement


## Robot Mechanism (Parallel Mechan ${ }^{\text {t }}$



Manipulators
Machine Tool Positioning Manipulator VR


Pipe Bender


Machine Tool

## Walking Assist Machines/Devices



Walking Chair


Water Surface Running Machine


Walking Assist Machine Using clutches


Joint Rehabilitation Mechanism

## Machine Elements/Manipulators $\star$



Rolling/Sliding Spherical Joints


Flexure Revolute Joint



Brakes for Robots


Under-actuated Manipulator

## Serial Mechanism and

## Parallel Mechanism



Serial Mechanism
All joints are active.


Parallel Mechanism
Only the prismatic joints are active.

## Parallel Mechanism



Typical Parallel Mechanism (Stewart-Gough platform)

Definition by Jean P. Merlet in "Parallel Robots": "A generalized parallel manipulator is a closed-loop kinematic chain mechanism whose end-effector is linked to the base by several independent kinematic chains", "A parallel robot is made up of an endeffector with $n$ degrees of freedom, and of a fixed base, linked together by at least two independent kinematic chains. Actuation takes place through $n$ simple actuators", "Parallel robots for which the number of chains is strictly equal to the number of degrees of freedom of the end-effector are called fully parallel manipulators."

Several kinematic structures (dof, arrangement of joints in kinematic chain, number of kinematic chains, etc)
Several applications: Machine tools, manipulators, coordinate measuring machine, motion simulator, positioning, etc.

## Fundamental Researches

## Transmission Index and Singular Point of Parallel Manipulators (1989-2000)

$\checkmark$ Transmission Index for Fully Parallel Manipulators(1993-1995)
$\checkmark$ Transmission Index for Wire-Driven Parallel Manipulators(1999)
$\checkmark$ Determination of Singular Point by Means of Transmission Index(1994)
$\checkmark$ Determination of Neighborhood/Vicinity of Singular Point(1995)

# Observation of Singular <br> Points(1989-1992) 



Motion in Working Space


Behavior around
Singular Points

## Concept of Pressure Angle $\mid$ |



A simple case of a single dof mechanism

Only the component of F along $\mathrm{V}, \mathrm{F} \cos \alpha$ can be transmitted to the object. So, basic idea is to make the angle $\alpha$ as close to 0 deg as possible. When $\alpha=90 \mathrm{deg}$, it corresponds to a singular configuration(point).

Transmission Index for a single dof mechanism: TI $=\cos \alpha$

## Transmission Index for

## Parallel Manipulators



## Definition of Transmission Index $T I$

$$
T I=\min \left(\left|\cos \alpha_{1}\right|, \cdots, \cos \left|\alpha_{N}\right|\right)
$$

Singular Point: $T I=0$

All of the inputs except for that of i-th connecting chain among $N$ chains of a parallel mechanism are locked. The direction of the force applied to the output link from each chain is geometrically determined. When the input joint of $i$-th chain is driven, the velocity and the force at the connection point form an angle as shown in the figure. This angle corresponds to the pressure angle. For $N$-dof mechanism, there exist $N$ pressure angles.

## Spherical Parallel mechanism wit High Motion Transmissibility (1993)

Tokyo Institute of Technology
Mechanical Systems Design Lab.


Overview of prototype(1993)

Motion Transmissibility and collision between the links were considered in the design

Swing angle : $\pm 65^{\circ} \sim 75^{\circ}$

## Repeatability:

$\pm 2 \sim 8 \mu \mathrm{~m}$ (arm length=200mm)
Absolute accuracy:0.043deg (Ave),
0.33 deg (worst)

Demonstration (Video)

## Reference

Yukio TAKEDA, Hiroaki FUNABASHI and Yasutaka SASAKI, Development of a Spherical In-parallel Actuated Mechanism with Three Degrees of Freedom with Large Working Space and High Motion Transmissibility (Evaluation of Motion Transmissibility and Analysis of Working Space), JSME International Journal, Series C, 393(1996,September), pp.541-548.

## Spatial Parallel mechanism with $\star$

## High Motion Transmissibility (1993)




TI vs. output position error (O:prototype)

Monte-Carlo technique was applied to the mechanism design of the prototype.

## Kinematic Calibration of

## Parallel Manipulator



Experimental Apparatus
Basement: $1,200 \times 1,200 \mathrm{~mm}^{2}$
Workspace : $650 \times 650 \times 350 \mathrm{~mm}^{3}$
$67 \times 60 \times 76 \mathrm{deg}^{3}$


Effect of calibration
Video

## Validation of Calibration



## References-Moion Transmissibility

1. Takeda, Y. and Funabashi, H., A Transmission index for in-parallel wiredriven mechanisms, JSME International Journal Series C, Vol. 44, No. 1, pp. 180-187, 2001.
2. Takeda, Y., Funabashi, H. and Ichimaru, H., Development of spatial inparallel actuated manipulators with six degrees of freedom with high motion transmissibility, JSME International Journal, Series C, Vol. 40, No. 2, pp. 299/308, 1997.
3. Takeda, Y. and Funabashi, H., Kinematic and static characteristics of inparallel actuated manipulators at singular points and in their neighborhoods, JSME International Journal, Series C, Vol. 39, No. 1, pp. 85-93, 1996.
4. Takeda, Y., Funabashi, H. and Sasaki, Y., Development of a spherical inparallel actuated mechanism with three degrees of freedom with large working space and High motion transmissibility (evaluation of motion transmissibility and analysis of working space), JSME International Journal, Series C, Vol. 39, No. 3, pp. 541-548, 1996.
5. Takeda, Y. and Funabashi, H., Motion transmissibility of in-parallel actuated manipulators, JSME International Journal, Series C, Vol. 38, No. 4, pp. 749755, 1995.

## Kinematic Design of Compensatable Parallel Manipulators

Yukio Takeda, Dr. Eng.

Professor, Dept .of Mechanical Sciences and Engineering
Director, Super-Mechano System Innovation \& Development Center
Tokyo Institute of Technology, Japan
htto://www.mech.titech.ac.jp/~msd/, htto://www.sms.titech.ac.jp/
Email: takeda@mech.titech.ac.jp
Presented at Robotics PPRIME Forum 2014, November 6, 2014, University of Poitiers, France

## Table of contents

1. Introduction
2. Kinematic design of parallel manipulator with redundant actuators: spatial six-dof parallel manipulator with redundant actuators for gross and fine motions
3. Kinematic design of lower-dof parallel mechanism to minimize the uncompensatable error: 3-URU pure rotational parallel mechanism
4. Kinematic design of lower-dof parallel mechanism with dof for compensating uncompensatable error
$\checkmark$ translational parallel manipulator with fine adjustment of platform orientation
$\checkmark$ two-dof rotational parallel mechanism with compensation for position error
$\checkmark$ rotational parallel mechanism with variable target point
5. Summary (Conclusions and future works)

## Introduction

Precise Manipulators:
$>$ with full dof(spatial: 6, planar: 3, spherical: 3)
$>$ with limited dof (lower dof) (pure rotational-motion/translational-motion manipulator)

Requirements:

- large stroke \& high resolution(accuracy)
- low parasitic motion

Problems:
$\square$ low dynamic range of actuator in displacement

- inevitable error of kinematic parameters and their effect on the output error
$\square$ umcompensatable error in limited-dof manipulator
Solutions:
$\checkmark$ use of redundancy for compensation with redundant actuators for gross and fine motions
$\checkmark$ use of redundancy for compensation with decoupled mechanism for limited-dof manipulator
$\checkmark$ kinematic design(optimization) based on the sensitivity


## Table of contents

1. Introduction
2. Kinematic design of parallel manipulator with redundant actuators: spatial six-dof parallel manipulator with redundant actuators for gross and fine motions
3. Kinematic design of lower-dof parallel mechanism to minimize the uncompensatable error: 3-URU pure rotational parallel mechanism
4. Kinematic design of lower-dof parallel mechanism with dof for compensating uncompensatable error
$\checkmark$ translational parallel manipulator with fine adjustment of platform orientation
$\checkmark$ two-dof rotational parallel mechanism with compensation for position error
$\checkmark$ rotational parallel mechanism with variable target point
5. Summary (Conclusions and future works)

## Application targets of

Applications
-Position adjusting machine for connection of optical fibers
-Fabrication machine of master disk for DVD -Micro/nano machines

## Required performance

Stroke : 50~100mm
In 3D space

## A solution to achieve

## a large stroke and fine resolution


gross motion drive (servo motor \& ball screw)

An example of a gross-fine drive for a single axis motion

## Our proposal:



## Advantages

- light inertia
- fast motion
- high precision
- high rigidity ect.

A redundant parallel manipulator with six-dof

## Basic Idea to Simultaneously Realize



Input-output relationship in
5-bar mechanism with two inputs and a single output (example) infinitesimal displacement

$$
\Delta \phi=\frac{c\left(\Delta l_{1}+\Delta l_{2} \cos \lambda\right)}{\cos \alpha}
$$

## Mechanism configuration



Redundant spatial parallel mechanism with six active joints for gross motion and six active joints for fine motion

## Our prototype(2001)



## Workspace

(T) $X, Y: \pm 40 \mathrm{~mm}$
$\mathrm{Z}: \pm 20 \mathrm{~mm}$
(R) $X, Y, Z: \pm 10^{\circ}$

## Desired resolution

Trans. : 20 nm
Rotat. : $0.2 \mu \mathrm{rad}$

## Size

$540 \mathrm{~mm} \times 540 \mathrm{~mm}$
$\times 600 \mathrm{~mm}^{3}$

## Prototype manipulator

## Workspace:

| Stroke | $12.3 \pm 3.5 \mu \mathrm{~m}$ |
| :--- | :--- |
| Applied voltage | 100 V |
| Size | $15.7 \times 15.7 \times 20 \mathrm{~mm}$ |

80(X) x 80(Y) x 40(Z) mm
20 deg (all direction)
Resolution:
20 nm (trans.), $0.2 \mu \mathrm{rad}$ (rotat.)
Performance of drive systems

|  | Gross motion | Fine motion |
| :--- | :---: | :---: |
| Stroke | 93 mm | $\pm 7 \mu \mathrm{~m}$ |
| Resolution | $0.2 \mu \mathrm{~m}$ | 4 nm |

## Gross-motion drive




Step response( $0.2 \mu \mathrm{~m}$ )
Resolution

$$
\begin{array}{cc}
\text { Realized } & \text { Design } \\
(0.2 \mu \mathrm{~m}) & (1 \mu \mathrm{~m})
\end{array}
$$

## Composition

(AC-Servo Motor \& Ball Screw of Lead 2.5mm)

## Fine-motion drive



Overview


Composition
ZT \& fiexure hinges)
Composition
(PZT \& flexure hinges)


Step response (4nm) Resolution
Realized < Design
(4nm)


Max. stroke

Stroke

> | Realized $=\begin{array}{c}\text { Design } \\ (10 \mu \mathrm{~m})\end{array}$ |
| :--- |

## Rolling spherical bearing



Composition

Specifications
(SRJ016T: Product of Hephaist Seiko Co, Ltd.)
Max. swing angle
$\pm 30 \mathrm{deg}$
Diameter of the sun ball
25.4 mm


Movie

## Control system of the prototype ${ }^{\text {® }}$

## manipulator



Composition of the control system


Composition of the fine-motion controller

## Inverse kinematics

Given : $p$ (position of the reference point and $T$ (orientation) of the output link

Obtain : input displacements $l_{1, i}$ and $l_{3, i}$ $(i=1,2, \ldots, 6)$ for gross and fine motions

## Closed loop equation:

$\overline{\mathrm{J}_{2, i} \mathrm{~J}_{\mathrm{P}, i}}=l_{3, i} \boldsymbol{e}_{3, i}=\boldsymbol{p}+T^{\mathrm{T}} \boldsymbol{p}_{i}-\boldsymbol{a}_{i}-l_{1, i} \boldsymbol{e}_{1, i}$
$T: 3 \times 3$ rotation matrix

$l_{3, i}^{2}=\boldsymbol{p}^{2}+\boldsymbol{p}_{i}^{2}+\boldsymbol{a}_{i}^{2}+l_{1, i}^{2}-2 \boldsymbol{p}_{i}^{\mathrm{T}} T\left(\boldsymbol{a}_{i}-l_{1, i} e_{1, i}\right)$


Vector representation of one connecting chain

This scalar equation has two unknown parameters (input displacements for gross and fine motions) in each chain for a specified pose of the output link.

## Inverse kinematics

## Closed loop equation:

$\overrightarrow{\mathrm{J}_{2, i} \mathrm{~J}_{\mathrm{P}, i}}=l_{3, i} \boldsymbol{e}_{3, i}=\boldsymbol{p}+T^{\mathrm{T}} \boldsymbol{p}_{i}-\boldsymbol{a}_{i}-l_{1, i} \boldsymbol{e}_{1, i}$
$T: 3 \times 3$ rotation matrix
Solution of input displacement $l_{3, i}$ : $l_{2}^{2}=\boldsymbol{p}^{2}+\boldsymbol{p}_{i}^{2}+\boldsymbol{a}_{i}^{2}+\left(l_{1, i}^{2}\right)-2 \boldsymbol{p}_{i}^{\mathrm{T}} T\left(\boldsymbol{a}_{i}-l_{1}, \boldsymbol{p}_{1, i}\right)$ $+2 p^{\mathrm{T}}\left(T^{\mathrm{T}} p_{i}-a_{i}-T_{1} . p_{1, i}\right)+2 T_{1, i} \boldsymbol{p}_{i}^{\mathrm{T}} e_{1, i}$
$l_{1, i}$ : predetermined standard value is given


Vector representation of one connecting chain
Solution of input displacement $l_{1, i, i}$ i

$$
\begin{aligned}
\left.l_{1 . i}^{2}\right) & +2\left(\boldsymbol{a}_{i}^{\mathrm{T}}-\boldsymbol{p}^{\mathrm{T}}-\boldsymbol{p}_{i}^{\mathrm{T}} T\right) \boldsymbol{e}\left(l_{1 . i}\right)+\boldsymbol{p}^{2}+\boldsymbol{p}_{i}^{2}+\boldsymbol{a}_{i}^{2} \\
& +2 \boldsymbol{p}^{\mathrm{T}}\left(T^{\mathrm{T}} \boldsymbol{p}_{i}-\boldsymbol{a}_{i}\right)-2 \boldsymbol{p}_{i}^{\mathrm{T}} T \boldsymbol{a}_{i}-l_{2 . i}^{2}=0 .
\end{aligned}
$$

## Control system of the prototype ${ }^{\text {® }}$

## manipulator



Composition of the control system


Composition of the fine-motion controller

## Experimental investigations

## Exp. 1 :Successive fine step motion around an initial pose

 was given as a desired output motion to investigate resolutionMeasurement of fine displacement:
Capacitive displacement sensor (ST-3512, Iwatsu Co., Ltd., measurement resolution : 6nm)


Change of desired pose with respect to time
Cases of experiments (Zc:central Z coordinate)

| No. | Initial position | Type of motion | Direction | Step magnitude |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $(0,0, \mathrm{Zc})[\mathrm{mm}]$ | Trans. | $Z$ | 10 nm |
| 2 | $(0,40, \mathrm{Zc})[\mathrm{mm}]$ | Rotat. | $Z$ | $0.5 \mu \mathrm{rad}$ |
| 3 | $(0,0, \mathrm{Zc}+20)[\mathrm{mm}]$ | Trans. | Y | 20 nm |
| 4 | $(0,0, \mathrm{Zc})[\mathrm{mm}]$ | Rotat. | X | $1 \mu \mathrm{rad}$ |

## Experimental investigations

## Exp. 1 :Successive fine step motion around a pose

Results of experiments:

(1) Z direction
translation, at $(0,0, \mathrm{Zc}) \mathrm{mm}$

(2) $Z$ direction, rotation, at $(0,40, \mathrm{Zc}) \mathrm{mm}$

It is known from these figures that successive step motions were successfully achieved for very fine steps.

## Experimental investigations

## Exp. 1 :Successive fine step motion around a pose

Results of experiments:

(3) Y direction
translation, at (0,0,Zc+20) mm

(4) $X$ direction,
rotation, at ( $0,0, \mathrm{Zc}$ ) mm

Summary of exp. 1 : fine translational and rotational motions were successfully achieved for very fine steps of 20 nm and $1 \mu$ rad in all directions and in the workspace.

## Experimental investigations

Exp. 2 :Successive fine step motion followed by a large displacement was given as the desired motion to investigate position repeatability in the motion with a large displacement


Fine step motion

## Experimental results:

Repeatability : $3 \sim 6 \mathrm{~nm}$ for $20 \sim 40 \mathrm{~mm}$ gross motion displacement and 50 nm fine motion successive steps

A redundant parallel manipulator with actuators for gross and fine motions was developed. Its control system was constructed and fine motion characteristics were experimentally investigated.

- A control system to achieve a fine motion together with a gross motion and a procedure to determine input displacement of gross and fine motion drive systems for a specified pose of the output link were presented.
- The positioning resolutions of our prototype manipulator were found to be 20 nm in the working space of $80 \times 80 \times 40 \mathrm{~mm}$.
- Position repeatability in fine positioning followed by a large displacement was also found to be better than 6 nm for a 20 or 40 mm displacement.

According to these results, a high ratio greater than $10^{6}$ of total stroke of gross motion to positioning resolution and repeatability was achieved by a parallel manipulator with redundant actuators for gross and fine motions.

Development of hybrid manipulator $\mid$ |


## Pure-rotational mechanism <br> -3-URU parallel mechanism

-Kinematic analysis and synthesis

- Error and stiffness analyses
- Calibration and compensation


## Pure-translational mechanism

-3-5R or 3-RUU parallel mechanism


Drawbacks of 6-dof mechanism such as Stewart-Gough platform with respect to workspace :
(1)Small orientation workspace
(2)Dependence of the orientation workspace on the position.


Considering such an application, rotational mechanism should have a large orientation workspace to have advantage compared with 6-dof mechanism.


3-RUU pure translational parallel mechanism


3-URU pure rotational parallel mechanism

## Accuracy/Error

|  | Output <br> displacement <br> error | Compensation <br> (by calibration, <br> feedback control) |
| :--- | :--- | :--- |
| 6-dof <br> mechanisms | 6-dimension | Compensatable |
| 3-dof spatial <br> mechanisms | 6-dimension | Partially (3 comp.) <br> compensatable <br> Partially (3 comp.) <br> uncompensatable |

These components should be considered in the mechanism design in order to achieve precise motion.

## Introduction (motivation)



Prototype of TPM (2007) (video)
Types of output pose error of a manipulator (caused by dimensional errors)
Compensatable error : it can be compensated for by calibration or full closed-loop control.

Uncompensatable error : it cannot be compensated for by any means. typical error in lower-dof manipulators.
Ex. Orientation error of the platform of a translational parallel manipulator

## Introduction (purpose)

Approaches to the uncompensatable error in limited(lower)-dof parallel mechanism :

1. To minimize uncompensatable error by determining the optimal values of kinematic constants at the design stage (Huda, Takeda, 2008), taking tolerances into account.
2. To change the structure of the mechanism so that fine adjustments can be added to the output motion to eliminate the uncompensatable error.

## Table of contents

1. Introduction
2. Kinematic design of parallel manipulator with redundant actuators: spatial six-dof parallel manipulator with redundant actuators for gross and fine motions
3. Kinematic design of lower-dof parallel mechanism to minimize the uncompensatable error: 3-URU pure rotational parallel mechanism
4. Kinematic design of lower-dof parallel mechanism with dof for compensating uncompensatable error
$\checkmark$ translational parallel manipulator with fine adjustment of platform orientation
$\checkmark$ two-dof rotational parallel mechanism with compensation for position error
$\checkmark$ rotational parallel mechanism with variable target point
5. Summary (Conclusions and future works)

## Target Application



Machine tool application of a pure rotational parallel mechanism

There are more applications of pure rotational and pure translational parallel mechanisms.

Requirements:

1. High Accuracy
2. Large Workspace
3. Etc.

This presentation is focused on the pure rotational parallel mechanism.

## Introduction (purpose)

## Approach to the uncompensatable error :

1. To minimize uncompensatable error by determining the optimal values of kinematic constants at the design stage (Huda, Takeda, 2008), taking tolerances into account.
2. To change the structure of the mechanism so that fine adjustments can be added to the output motion to eliminate the uncompensatable error.


Drawbacks of 6-dof mechanism such as Stewart-Gough platform with respect to workspace :
(1)Small orientation workspace
(2)Dependence of the orientation workspace on the position.


Considering such an application, rotational mechanism should have a large orientation workspace to have advantage compared with 6-dof mechanism.

## Purpose of the present work

## and composition of the presentation

Purpose : to design a pure rotational parallel mechanism performing precise motion within a large workspace. (3-URU structure is considered)

Table of contents:

1. Mechanism configuration and basic design flow
2. Error analysis and the uncompensatable error index
3. Result of design and prototype
4. Conclusions

## Mechanism configuration



3-URU pure rotational parallel mechanism (a special case of 3-5R parallel mechanism)

Condition for pure rotational motion:

1. The first and fifth axes of each chain meet at the point, which is the center of rotation of the platform.
2. The $2^{\text {nd }}, 3^{\text {rd }}$ and $4^{\text {th }}$ joint axes of each chain are parallel.

Kinematic constants:
$r_{\mathrm{B}}, r_{\mathrm{P}}$ : location radii of the universal joints on the base and the platform
$\psi$ : angle of the first joint axis from the base plane
$\zeta$ : angle of the fifth joint axis from the platform plane
$L_{2}, L_{3}$ : link lengths
These are determined in the design while the above conditions are satisfied.

## Mechanism configuration

## (3-URU mechanism) Condition for pure rotational motion:

1. The first and fifth axes of each chain meet at the point, which is the center of rotation of the platform.
2. The $2^{\text {nd }}, 3^{\text {rd }}$ and $4^{\text {th }}$ joint axes of each chain are parallel.

Kinematic constants:
$r_{\mathrm{B}}, r_{\mathrm{p}}$ : location radii of the universal joints on the base and the platform
$\psi$ : angle of the first joint axis from the base plane
$\zeta$ : angle of the fifth joint axis from the platform plane
$1^{\text {st }}$ prototype of 3 -URU pure rotational parallel mechanism (a special case of $3-5 R$ parallel mechanism)
$L_{2}, L_{3}$ : link lengths
These are determined in the design while the above conditions are satisfied.

## Basic design flow

## (singularity and workspace)

Condition for basic design:


3-URU mechanism
(1) The reachable workspace by the mechanism should include the prescribed workspace.
(2) There are no singular points in the prescribed workspace.

Basic design flow (Huda and Takeda,2007):
(1) Evaluation of kinematic constants $\psi, \zeta, r_{\mathrm{B}}$ and $r_{\mathrm{p}}$ based on singularity conditions.
(2) Evaluation of kinematic constants other than $\psi, \zeta, r_{\mathrm{B}}$ and $r_{\mathrm{p}}$ taking account of the reachable workspace.
(3) Optimization and determination of kinematic constants using a performance index.
As the performance index, we use the uncompensatable error index in this work.

## Error model



## Types of error:

- Errors in parallel axes between $R_{2}$ and $R_{3}$ and between $R_{3}$ and $R_{4}$.
- Errors in intersecting axes between $R_{1}$ and $R_{2}$ and between $R_{4}$ and $R_{5}$.
- Angular errors of the universal joints on the base and platform.
- Position errors of the universal joints on the base and platform.


## Introduction of a virtual joint

## to represent a kinematic error



Model of angular error


Model of linear error

## Sensitivity formulation and

## the Uncompensatable error

Sensitivity equation for $l$ error sources:

$$
\Delta \boldsymbol{X}_{\text {total }}=\sum_{m=1}^{l} \Delta \boldsymbol{X}_{m}=\sum_{m=1}^{l}\left(J_{x n}\right)^{-1} J_{q m} \Delta \boldsymbol{q}_{m}=\sum_{m=1}^{l} S_{m} \Delta \boldsymbol{q}_{m}
$$



This matrix can be obtained by the reciprocal screw theory.
Uncompensatable error of a pure rotational parallel mechanism: Position part of the above equation is related to the uncompensatable error. This means the position error of the center of rotation of the platform cannot be compensated by calibration nor closed-loop control.

##  Mechanical Systems Design Lab. <br> the Uncompensatable error index

Uncompensatable error index:
$\Delta E_{\text {max }}$ is the maximum position error taking all error sources into consideration under the following condition. $\Delta q_{m}$ is determined based on the manufacturing tolerance.

$$
\Delta \boldsymbol{x}_{m}=S_{\mathrm{T}_{m}} \Delta \boldsymbol{q}_{m} \quad\left[\begin{array}{l}
-1 \\
-1 \\
-1
\end{array}\right] \Delta q_{m} \leq \Delta \boldsymbol{q}_{m} \leq\left[\begin{array}{l}
1 \\
1 \\
1
\end{array}\right] \Delta q_{m}
$$

## Optimal design



Machine tool application of a pure rotational parallel mechanism
In this application, two angles $\left(\xi_{x}, \xi_{z}\right)$ are necessary to control. The resultant angle $\phi$ is considered redundant. This redundant orientation is optimized to minimize the uncompensatable error for each set $\left(\xi_{x}, \xi_{z}\right)$.


Orientation angles of the platform
Design specifications:

- orientation workspace

$$
\left.\begin{array}{l}
0 \leq \xi_{x} \leq \xi_{x(\max )} \\
0 \leq \xi_{z} \leq 2 \pi
\end{array}\right\}
$$

- position of the center of rotation relative to the platform


## Optimal design

Optimal design was carried out to reveal the characteristics of the 3-URU mechanism. The results were summarized by design charts to represent the following relationships.

1. The relationship between the maximum inclination angle and the optimal kinematic constants, while the position of the center of rotation is kept constant.
2. The relationship between the position of the center of rotation and the optimal kinematic constants, while the maximum inclination angle is kept constant.
*In this optimal design, the optimization process of the redundant orientation angle $\phi$ is included.

## Optimal design result(1)



Relationship between $\xi_{x(\max )}$ and the optimal kinematic constants and $\operatorname{Max}\left(\Delta E_{\max }\right)$ at $\zeta=0$
It is known from the figure that for a large inclination angle such as $\xi_{x(\max )}>1$ rad, smaller base radius relative to the platform results in lower sensitivity with respect to uncompensatable error. Moreover, $\underline{\operatorname{Max}\left(\Delta E_{\max }\right) \text { increases }}$ with the maximum inclination angle.

## Optimal design result(2-a)



It is known from the figyre that for smaller inclination angle there are some ranges of $\zeta$ by which the uncompensatable error is kept within a small range ( $-0.4 \leq \zeta \leq 0.4 \mathrm{rad}$ ).

## Optimal design result(2-b)


(b) $\xi_{\mathrm{x}(\text { max })}=1.3 \mathrm{rad}$

Relationship between $\zeta$ and optimal kinematic constants and $\operatorname{Max}\left(\Delta E_{\max }\right)$

It is known from the figure that there are few range of $\zeta$ that give small uncompensatable error for a larger inclination angle. It is also found that the best choice for this case is expected to be $\zeta=0$.

## Prototype



Overview of the prototype


Configuration at $\xi_{x}=65^{\circ}$ (with the co-authors)

## Conclusions-opiimal desion for ow sensitivit

In the present paper, we carried out a kinematic design of 3-URU pure rotational parallel mechanism with a large workspace subject to minimization of the position error of the center of platform rotation.
(1) Optimal kinematic constants and the magnitude of the position error of the center of platform rotation for the design specification in the maximum inclination angle of the platform have been clarified. The result was summarized in some charts that are useful in kinematic design.
(2) Using the charts, it was found that the center of platform rotation should be located on the platform plane for a large maximum inclination angle. On the other hand, the center of platform rotation can be specified within some range considering the application of the mechanism for a small maximum inclination angle.
(3) An optimal design result and a prototype was shown for an orientating device of a machine tool which can achieve a large maximum inclination angle such as 1.3 rad based on the results mentioned above.

## Table of contents

1. Introduction
2. Kinematic design of parallel manipulator with redundant actuators: spatial six-dof parallel manipulator with redundant actuators for gross and fine motions
3. Kinematic design of lower-dof parallel mechanism to minimize the uncompensatable error: 3-URU pure rotational parallel mechanism
4. Kinematic design of lower-dof parallel mechanism with dof for compensating uncompensatable error
$\checkmark$ translational parallel manipulator with fine adjustment of platform orientation
$\checkmark$ two-dof rotational parallel mechanism with compensation for position error
$\checkmark$ rotational parallel mechanism with variable target point
5. Summary (Conclusions and future works)

## Introduction (motivation)



Prototype (2007)
(video)
Types of output pose error of a manipulator (caused by dimensional errors)
Compensatable error : it can be compensated for by calibration or full closed-loop control.

Uncompensatable error : it cannot be compensated for by any means. typical error in lower-dof manipulators.
Ex. Orientation error of the platform in a translational parallel manipulator

## Introduction (purpose)

## Approach to the uncompensatable error :

1. To minimize uncompensatable error by determining the optimal values of kinematic constants at the design stage (Huda, Takeda, 2008), taking tolerances into account.
2. To change the structure of the mechanism so that fine adjustments can be added to the output motion to eliminate the uncompensatable error.

The present paper discusses the kinematic design of a translational parallel manipulator with fine adjustment capability of platform orientation (TPMFAO).

1. Introduction
2. Basic concept underlying the structural synthesis of TPMFAO
3. Review of the kinematic structures for translational and rotational parallel mechanisms with three dof.
4. Derivation for the kinematic structures of TPMFAO.
5. Design of a prototype manipulator and its orientation compensation capability using experimental results.
6. Conclusions.

## Basic concept

## for structural synthesis of TPMFAO

Target manipulator : A manipulator must have six degrees of freedom to achieve fine adjustments of the platform orientation with gross translational motion.
Key point : output motion = main (translation) + sub (rotation) each with 3 dof

Conditions considered in the structural synthesis:
(1) It has three connecting chains.
(2) Each connecting chain has the same structure.
(3) Revolute and prismatic joints are used to compose a connecting chain.
(4) Each connecting chain has two active joints. One active joint is to correspond to the translational output motion, and is called a main active joint. The other is used to make fine adjustment to the platform's orientation, and is a sub-active joints.
(5) The main active joints are located at the first or second joints, while any location for the sub-active joints is accepted.

## Candidates for TPMFAO

Fully-decoupled mechanism


Partially-decoupled mechanism


Input-output infinitesimal displacement relationships:

$$
\begin{array}{ll}
{\left[\begin{array}{l}
\boldsymbol{\Delta} \boldsymbol{\theta} \\
\boldsymbol{\Delta} \boldsymbol{X}
\end{array}\right]=J_{\mathrm{FD}}\left[\begin{array}{l}
\boldsymbol{\Delta} \boldsymbol{q}_{\mathrm{M}} \\
\boldsymbol{\Delta} \boldsymbol{q}_{\mathrm{S}}
\end{array}\right]} & {\left[\begin{array}{l}
\boldsymbol{\Delta} \Theta \\
\boldsymbol{\Delta} \boldsymbol{X}
\end{array}\right]=J_{\mathrm{PD}}\left[\begin{array}{l}
\boldsymbol{\Delta} \boldsymbol{q}_{\mathrm{M}} \\
\boldsymbol{\Delta} \boldsymbol{q}_{\mathrm{S}}
\end{array}\right]} \\
J_{\mathrm{FD}}=\left[\begin{array}{cc}
0_{3} & B_{\mathrm{FD}} \\
A_{\mathrm{FD}} & 0_{3}
\end{array}\right] & J_{\mathrm{PD}}=\left[\begin{array}{ll}
0_{3} & B_{\mathrm{PD}} \\
A_{\mathrm{PD}} & C_{\mathrm{PD}}
\end{array}\right]
\end{array}
$$

## Fully-decoupled mechanism

## for TPMFAO (Synthesis conditions)

Conditions for chain:
Each connecting chain should become
(1) a chain for a translational parallel mechanism when the subactive joints are locked at any position, and
(2) a chain for a pure rotational parallel mechanism when the main active joints are locked at any position.

Conditions for joint :
(1) Conditions for generating translational motion
i. The axis directions of revolute joints are two.
ii. There are fewer than four prismatic joints.
(2) Conditions for generating pure rotational motion
i. There are more than one revolute joint belonging to the first group.
ii. Prismatic joints are perpendicular to the revolute joints belonging to the second group.
iii. There are fewer than three prismatic joints.

## Fully-decoupled mechanism

## for TPMFAO (Synthesis results)

| Classes | Types of connecting chains |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5R1P | $\mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{15 \mathrm{sa}} \mathrm{R}_{1}$ | $\mathrm{R}_{2} \mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{15 \mathrm{sa}} \mathrm{R}_{1}$ | $\mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{1} \mathrm{R}_{1}{ }_{\text {sa }}$ | $\mathrm{R}_{2} \mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{1} \mathrm{R}_{1 \text { sa }}$ |  |  |
| 4R2P | $\begin{aligned} & \dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{P}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{15 \mathrm{sa}} \mathrm{R}_{1} \\ & \dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{P}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{1} \mathrm{R}_{1 \mathrm{sa}} \end{aligned}$ | $\begin{aligned} & \overline{\mathrm{P}} \mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{15 \mathrm{~s}} \mathrm{R}_{1} \\ & \overline{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \mathrm{R}_{1} \mathrm{R}_{1 \mathrm{sa}} \end{aligned}$ | $\begin{aligned} & \dot{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \overline{\mathrm{P}}_{2} \mathrm{R}_{15 \mathrm{sa}} \mathrm{R}_{1} \\ & \dot{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \overline{\mathrm{P}}_{2} \mathrm{R}_{1} \mathrm{R}_{1 \mathrm{sa}} \end{aligned}$ | $\begin{aligned} & \mathrm{R}_{2} \dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{P}} \mathrm{R}_{2} \mathrm{R}_{15 \mathrm{sa}} \mathrm{R}_{1} \\ & \mathrm{R}_{2} \dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{P}}_{2} \mathrm{R}_{1} \mathrm{R}_{1 \mathrm{sa}} \end{aligned}$ | $\begin{aligned} & \dot{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \overline{\mathrm{PR}}_{15 \mathrm{sa}} \mathrm{R}_{1} \\ & \dot{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{R}_{2} \overline{\mathrm{PR}}_{1} \mathrm{R}_{1 \mathrm{sa}} \end{aligned}$ | $\begin{aligned} & \mathrm{R}_{2} \mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \overline{\mathrm{P}}_{1 \mathrm{sas}} \mathrm{R}_{1} \\ & \mathrm{R}_{2} \mathrm{P}_{\mathrm{ma}} \mathrm{R}_{2} \overline{\mathrm{P}}_{1} \mathrm{R}_{1 \mathrm{sa}} \end{aligned}$ |
| 3R3P | $\dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{P} P} \tilde{P_{2}} \mathrm{R}_{1} \mathrm{R}_{1}$ <br> $\operatorname{PPPR}_{1} \mathrm{R}_{1} \mathrm{R}_{1}$ | $\overline{\mathrm{P}}_{\mathrm{ma}} \tilde{\mathrm{Pr}}_{2} \mathrm{R}_{1} \mathrm{R}_{1}$ | $\dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{PR}}_{2} \tilde{\mathrm{PR}}_{1} \mathrm{R}_{1}$ | $\overline{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \mathrm{PR}_{1} \mathrm{R}_{1}$ | $\dot{\mathrm{P}}_{\mathrm{ma}} \mathrm{R}_{2} \overline{\mathrm{P}} \tilde{\mathrm{PR}}_{1} \mathrm{R}_{1}$ | $\mathrm{R}_{2} \mathrm{P}_{\mathrm{ma}} \overline{\mathrm{P}} \tilde{\mathrm{PR}}_{1} \mathrm{R}_{1}$ |

(Subscript "ma" means that it must be a main active joint.)

$\mathrm{R}_{2} \dot{\mathrm{P}}_{\mathrm{ma}} \overline{\mathrm{P}} \widetilde{\mathrm{P}} \mathrm{R}_{1} \mathrm{R}_{1}$ connecting chain for full-decoupled mechanism

## 風

Tokyo Institute of Technology Mechanical Systems Design Lab.

## for TPMFAO (Synthesis resulis)

| Classes | Types of connecting chains |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6R | $\mathrm{R}_{\text {sa }} \mathrm{RRR} \times \overline{\mathrm{RR}}$ | $\mathrm{RRRR}^{\text {sa }}$ ( $\overline{\mathrm{RR}}$ | $\dot{\mathrm{RRR}} \overline{\mathrm{RRR}}^{\text {sa }}$ |  |  |  |  |  |
| 5R1P | $\begin{aligned} & \mathrm{R}_{\mathrm{sa}} \mathrm{RR} \overline{\mathrm{RR} P} \\ & \dot{R R P R}_{\mathrm{sa}} \overline{\mathrm{RR}} \\ & \text { RRRRPR }_{5 \mathrm{R}} \overline{\mathrm{R}} \\ & \mathrm{R}_{\mathrm{sa}} \mathrm{PRRRR} \overline{\mathrm{R}} \end{aligned}$ | $\dot{R R R}_{\mathrm{sa}} \overline{\mathrm{RR} P}$ <br> $\dot{\mathrm{RRP}} \overline{\mathrm{RRR}}_{\mathrm{sa}}$ <br> $\dot{R}^{\text {RRRPR }} \bar{R}_{\text {sa }}$ <br> $\mathrm{PR}_{\mathrm{sa}} \mathrm{RRR} \overline{\mathrm{R}}$ |  | RRं $\overline{\mathrm{RRP}} \mathrm{R}_{\mathrm{sa}}$ <br> $\dot{R R R R}_{5 \mathrm{~s}} \overline{\mathrm{R}} \mathrm{P}$ <br> $\operatorname{RRPRR}_{\mathrm{sa}} \overline{\mathrm{R}}$ <br> PRRR $\bar{R}_{\text {sa }}$ | $\begin{aligned} & \dot{\mathrm{RRR}}_{\mathrm{sa}} \overline{\mathrm{R}} \mathrm{P} \overline{\mathrm{R}} \\ & \mathrm{RRR}_{\mathrm{R}}^{\mathrm{R}} \mathrm{R}_{\mathrm{sa}} \mathrm{P} \\ & \dot{\mathrm{RR} P \mathrm{R}} \mathrm{R}_{\mathrm{sa}} \end{aligned}$ | $\dot{\mathrm{R} R} \overline{\mathrm{R}} \mathrm{P} \overline{\mathrm{R}} \mathrm{R}_{\mathrm{sa}}$ <br> RRR $\bar{R} P_{\text {sa }}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{RPRR} \overline{\mathrm{R}}$ | $\begin{aligned} & \mathrm{R}_{\mathrm{sa}} \mathrm{RRP} \overline{\mathrm{RR}} \\ & \mathrm{R}_{\mathrm{sa}} \mathrm{RRRPP} \overline{\mathrm{R}} \\ & \operatorname{RPRRR}_{\mathrm{sa}} \overline{\mathrm{R}} \end{aligned}$ | $\begin{aligned} & \dot{\mathrm{RRR}}_{5 \mathrm{sa}} \mathrm{P} \overline{\mathrm{RR}} \\ & \mathrm{R}^{\mathrm{RRR}} \mathrm{R}_{\mathrm{sa}} \mathrm{P} \overline{\mathrm{R}} \\ & \dot{\mathrm{RPRR}} \overline{\mathrm{R}}_{\mathrm{sa}} \end{aligned}$ |
| 4R2P | $\mathrm{R}_{\mathrm{sa}} \dot{\mathrm{R}} \dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{P}$ <br> $\dot{\operatorname{RR}} \overline{\mathrm{R}} \overline{\mathrm{R}}_{\mathrm{sa}} \mathrm{P}$ <br> $\operatorname{PRRR}_{\mathrm{sa}} \overline{\mathrm{R}} \mathrm{P}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{R} P \mathrm{PRP} \overline{\mathrm{R}}$ <br> $\mathrm{PRR}_{\mathrm{R}} \overline{\mathrm{R}}_{\mathrm{sa}}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{PPRR} \overline{\mathrm{R}}$ |  | $\dot{R} \dot{R} \bar{R} R_{\text {sa }} P P$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{RPR} \overline{\mathrm{R}} \mathrm{P}$ <br> PRR $\bar{R}{ }^{2} R_{\text {sa }}$ <br> RPR $^{2} P_{59} \bar{R}$ <br> $\operatorname{RPPRR}_{\text {sa }} \bar{R}$ <br> $\mathrm{PPR}_{\mathrm{sa}} \mathrm{R} \mathrm{R} \overline{\mathrm{R}}$ | $\dot{\mathrm{R} R} \overline{\mathrm{R}} \mathrm{PR}_{\mathrm{sa}} \mathrm{P}$ <br> $\dot{R P R R}_{\text {sa }} \overline{\mathrm{R}} \mathrm{P}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{RRPP} \overline{\mathrm{R}}$ <br>  <br> $\dot{R}^{\text {RPPR }} \overline{\mathrm{R}}_{\mathrm{sa}}$ <br> $\operatorname{PPRRR}_{5 \mathrm{~s}} \overline{\mathrm{R}}$ | $\dot{R} \dot{R} \bar{R} P_{P R}$ <br> $\dot{R P R} \bar{R} R_{s a} P$ <br> $\dot{R}^{R} R_{\text {sa }} P P \bar{R}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{PRR} \mathrm{RP} \overline{\mathrm{R}}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{PRPR} \overline{\mathrm{R}}$ <br> PPRR $^{\operatorname{R}} \mathrm{R}_{\mathrm{sa}}$ | $\mathrm{R}_{\mathrm{sa}} \mathrm{R} \dot{R} \overline{\mathrm{R}} \mathrm{P}$ <br> $\dot{R} P \dot{R} \bar{R} P_{\text {sa }}$ <br> $\dot{R}^{R}$ PR $_{59} P \bar{R}$ <br> $\mathrm{PR}_{\text {sa }} \mathrm{RR} P \bar{R}$ <br> $\mathrm{PR}_{\mathrm{se}} \mathrm{R} P \mathrm{R} \overline{\mathrm{R}}$ | $\begin{aligned} & \dot{R R R}_{5 a} \overline{\mathrm{P}} \overline{\mathrm{R}} \\ & \mathrm{R}_{\mathrm{sa}} \mathrm{PRR} \overline{\mathrm{R}} \overline{\mathrm{R}} \\ & \mathrm{RRPPR}_{\mathrm{sa}} \overline{\mathrm{R}} \\ & \mathrm{PR}^{2} \mathrm{RR}_{\mathrm{sa}} \overline{\mathrm{R}} \\ & \mathrm{PRPRR}_{\mathrm{sa}} \overline{\mathrm{R}} \end{aligned}$ |  |
| 3R3P | $\mathrm{R}_{\mathrm{sa}} \dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{PPP}$ <br> $\dot{R}^{\mathrm{R}} \mathrm{R}_{\mathrm{sa}} \overline{\mathrm{R}} \mathrm{PP}$ <br> $P R \bar{R} P_{\text {sa }} P$ <br> $\mathrm{R}_{5 \mathrm{~s}} \mathrm{PRP} \overline{\mathrm{R}} \mathrm{P}$ | $\dot{R}_{\mathrm{sa}} \overline{\mathrm{R}} \mathrm{PPP}$ <br> $\dot{\mathrm{R} P} \overline{\mathrm{R}}_{\mathrm{sa}} \mathrm{PP}$ <br> $P \dot{R} \bar{R} P P R_{s a}$ <br> $P_{\text {sa }} \mathrm{R} P \bar{R} P$ | $\begin{aligned} & \hline \dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{R}_{\mathrm{sa}} \mathrm{PPP} \\ & \dot{\mathrm{R} P \bar{R} P R_{\mathrm{sa}} \mathrm{P}} \\ & \mathrm{R}_{\mathrm{sa}} \dot{\mathrm{R} P P} \overline{\mathrm{R}} \mathrm{P} \\ & \mathrm{PR}_{\mathrm{sa}} \mathrm{P} \overline{\mathrm{R}} \mathrm{P} \end{aligned}$ | $\dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{PR}_{\mathrm{sa}} \mathrm{PP}$ <br> $\dot{\mathrm{R}} \mathrm{P} \overline{\mathrm{R}} \mathrm{PPR}_{\mathrm{sa}}$ <br> $\dot{R}_{\mathrm{sa}} \mathrm{PP} \overline{\mathrm{R}} \mathrm{P}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{RPPPP} \overline{\mathrm{R}}$ | $\dot{R} \bar{R} P_{P R}{ }_{51}$ <br> $\mathrm{R}_{\mathrm{sa}} \mathrm{PR} \overline{\mathrm{R}} \mathrm{PP}$ <br> $\dot{R}^{-1} R_{\text {sa }} P \bar{R} P$ <br> $\dot{R}_{\text {sa }} P P P \bar{R}$ | $\dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{PPPR}_{\text {sa }}$ <br> $\mathrm{PR}_{\mathrm{sa}} \mathrm{R} \overline{\mathrm{R}} \mathrm{PP}$ <br> RPPR $_{\text {sa }} \overline{\mathrm{R}} \mathrm{P}$ <br> R $^{2} R_{59} P \bar{R} \bar{R}$ | $\begin{aligned} & \mathrm{R}_{\mathrm{sa}} \dot{\mathrm{R} P \overline{\mathrm{R}} P \mathrm{P}} \\ & \mathrm{PRR}_{\mathrm{sa}} \overline{\mathrm{R}} \mathrm{PP} \\ & {\dot{\mathrm{R} P P} \overline{\mathrm{R}}_{\mathrm{sa}} \mathrm{P}}^{2} \end{aligned}$ | $\dot{R}_{\text {sa }} \mathrm{P} \overline{\mathrm{R}} \mathrm{PP}$ <br> $\mathrm{PR} \overline{\mathrm{R}} \mathrm{R}_{\mathrm{sa}} \mathrm{PP}$ <br> $\dot{\mathrm{R} P P} \overline{\mathrm{R}} \mathrm{PR}_{\mathrm{sa}}$ |

(Subscript "sa" means that it must be a sub-active joint.)

$\mathrm{R}_{\mathrm{sa}} \dot{R} \dot{R} \dot{R} \overline{\mathrm{R} R}$ connecting chain
for partially-decoupled mechanism

## Kinematic design of

## prototype manipulator

Mechanism configuration :


Kinematic constants of the R RRRRR parallel mechanism

## Target application : assembly

Required characteristics:
(1) large workspace
(2) high precision
i. fine orientation-adjustment capability
ii. small coupled translational motion with rotational motion by sub-input

partially-decoupled

## Kinematic design of

Performance indices :

1. Singularity and utility workspace volume constraint singularity $\longrightarrow$ angle $\beta=\pi / 4$ normalized volume index

$$
N V I=\text { volume of the utility workspace } /\left(2 \pi L^{3} / 3\right)
$$

2. Orientation-adjustment capability ( $O A C$ ) the maximum singular value of $B_{\mathrm{PD}}$ of $J_{\mathrm{PD}} \quad J_{\mathrm{PD}}=\left[\begin{array}{cc}0_{3} & B_{\mathrm{PD}} \\ A_{\mathrm{PD}} & C_{\mathrm{PD}}\end{array}\right]$
3. Coupling index (CI) the maximum singular value of $C_{\mathrm{PD}} B_{\mathrm{PD}}^{-1}$

## Kinematic design of

## prototype manipulator

Result :

$$
\begin{aligned}
& r_{\mathrm{b}}=0.22 \mathrm{~m}, r_{\mathrm{p}}=0.086 \mathrm{~m}, d_{1}=0.06 \mathrm{~m}, a_{2,3}=a_{3,4}=0.115 \mathrm{~m}, \\
& a_{4,5}=0, a_{5,6}=0.19 \mathrm{~m}, \beta=\pi / 4
\end{aligned}
$$



Distributions of evaluation indices in the reachable workspace

## Prototype manipulator and

 experiments

Prototype manipulator (video of motion)

## Prototype manipulator and

## experiments


( $X$ : from 0 to $0.11 \mathrm{~m}, Y=0, Z=0.35 \mathrm{~m}$ )

( $X$ : from 0 to $-0.12 \mathrm{~m}, Y=0, Z=0.35 \mathrm{~m}$ )

## Experimental results

Observations:
(1) Orientation error is large at points of large Cl . Then, it is expected that a mechanism with small orientation error without orientation adjustment by the sub-active joints can be designed using Cl as one of the evaluation indices in the kinematic synthesis.
(2) The orientation error of the platform was reduced by using the sub-active joints. (video of compensation motion)

## Conclusions-TPMFAO

We presented a kinematic design of translational parallel manipulator with fine adjustment capability of platform orientation (TPMFAO).

1. To clarify all possible kinematic structures whose main motion is translation and sub-motion is rotation, we carried out structural synthesis of fully and partially decoupled mechanisms based on the synthesis results of 3-dof translational and rotational parallel mechanisms. As the result, we obtained 129 structures.
2. Based on the result in 1, we designed and built a prototype manipulator, and its basic characteristics were shown and discussed. It is known from the results that the coupling index is an appropriate index for the kinematic design of translational parallel manipulators for the mechanism with small orientation error without compensation.
3. Orientation adjustment of the platform by the sub-inputs of the TPMFAO was successfully achieved.

## Table of contents

1. Introduction
2. Kinematic design of parallel manipulator with redundant actuators: spatial six-dof parallel manipulator with redundant actuators for gross and fine motions
3. Kinematic design of lower-dof parallel mechanism to minimize the uncompensatable error: 3-URU pure rotational parallel mechanism
4. Kinematic design of lower-dof parallel mechanism with dof for compensating uncompensatable error
$\checkmark$ translational parallel manipulator with fine adjustment of platform orientation
$\checkmark$ two-dof rotational parallel mechanism with compensation for position error
$\checkmark$ rotational parallel mechanism with variable target point
5. Summary (Conclusions and future works)

## Background

- Demand


## Machine tools for 3D-shaped objects

Aspherical lens, objects with free-form surfaces

- Required characteristics
- Ability to change the tool orientation around two axes within a large range of motion
- Ability to precisely keep the position of the center of rotation of the tool and orientation of the tool
- Decoupled motion(translation and rotation)

Under the conditions of
$\checkmark$ Large machining load
$\checkmark$ No physical supporting elements at TP

## Configuration of proposed machine tool



Machine tool using rotational parallel mechanism

## Configuration of proposed machine tool



## Features

$\checkmark$ RPM performs 2-dof rotational motion around TP
$\checkmark$ No physical supporting elements at TP
$\checkmark$ Ability to manufacture objects with free-form surfaces, including concave and convex surfaces

Center of rotation of tool (TP)

Machine tool using rotational parallel mechanism

## Purpose of research

## Problems to be solved (focused on RPM)

$\checkmark$ To precisely keep the position of TP
without physical supporting elements at TP
$\checkmark$ To achieve a large orientation around 2 axes

## Approach

$\checkmark$ Structural and dimensional synthesis of RPM
A mechanism that theoretically achieves rotational output motion around two axes can be synthesized by considering the constraints by the connecting chains imposed on the output link. However,


Configuration of proposed machine tool

## Consideration to the position error of TP (uncompensatable error by RPM)

Position error of TP, that is caused by manufacturing and assembly errors and elastic deformation of parts, cannot be compensated for by RPM itself.

## Purpose of research

Structure and dimensions of mechanism, that can perform 2dof translational motion in the output link's plane as well as 2dof rotational motion, is proposed.


Configuration of proposed machine tool

## Related works

- Rotational parallel mechanism
$\checkmark$ Structural synthesis (Kong, 2004) (Karouia, 2005)
$\checkmark$ Dimensional synthesis taking into consideration workspace, singularity, and motion transmissibility (Takeda, 1996), (Huda, 2007)
- Uncomsatable error (parasitic motion) of lower-dof PM
$\checkmark$ Kinematic synthesis of 3-URU pure RPM based on sensitivity analysis(Huda, 2008)
$\checkmark$ Kinematic calibration of a translational PM (Huang, 2003)
$\checkmark$ Kinematic design of translational PM with orientation error compensation function (Tanabe, 2010)

However, we have not found any rotational mechanisms that achieve precise positioning of the target point within a large workspace or any method for designing such a mechanism.

## Table of contents

1. Introduction
2. Structural Synthesis

Basic condition
Function of connecting chains
Structural synthesis of constraint chain
Structural synthesis of actuation chain
3. Dimensional Synthesis

Evaluation indices
Determination of kinematic constants
4. Experiments

Design and fabrication of prototype
Experiments
5. Conclusions

## 2. Structural Synthesis

## Basic condition

(1) Connecting chains are composed of prismatic and revolute joints.
(2) Overconstrained, redundant, and redundant actuation mechanisms are not considered.
(3) There are three connecting chains. This condition was determined by considering the stiffness characteristics of the mechanism in all directions, avoidance of collisions between links, and cost reduction.
(4) The mechanism has four DOF. Two of them are used for orientation control of the output link within a large workspace, and the other two are used for fine compensation for position error of the target point.
(5) Two of the three connecting chains have the same structure and kinematic constants.

## 2. Structural Synthesis

## Function of connecting chains

- Required output motion (total 4 dof):
$\checkmark$ rotational motion with 2 dof as the main output motion
$\checkmark$ translational motion with 2 dof as the compensation motion
- Classification and function of connecting chains
- Constraint chain (one)
(1)composed of passive joints
(2)constraint force perpendicular to the output link's plane and passing through TP
(3) constraint moment around 1 axis
- Actuation chains (two) (for each)
(1)two active joints
(2)one active joint corresponds to rotational output motion
(3)the other active joint corresponds to translational output motion


## 2. Structural Synthesis

## Result



## 3. Dimensional Synthesis

## Evaluation indices

(1) For output velocity characteristics
(2) For torque transmissibility
(3) For decoupling characteristics in compensation motion

Synthesis procedure composed of two steps
(1) Determination of kinematic constants regarding rotational output motion based on indices (1) and (2)
(2) Determination of kinematic constants regarding compensation motion based on indices (3)

## 3. Dimensional Synthesis

Determination of kinematic constants regarding rotational output motion


Symmetrical arrangement of three chains


Related design parameters

Orthogonal arrangement of actuation chains
Evaluation indices: $I_{\mathrm{A}}$ and $I_{\mathrm{S}}$

## 3. Dimensional Synthesis

Determination of kinematic constants regarding compensation motion



Type B (prismatic joint) Related design parameters

Evaluation index: $C I$

## 3. Dimensional Synthesis

Result (In the figure, the other actuation chain is not shown.)


## 4. Experiments

Design and fabrication of prototype


Overview of prototype
$\checkmark$ Collision between links $\checkmark$ Stiffness of links and mechanism

## 4. Experiments

Experiment (workspace: rotational motion)


## 4. Experiments

Experiment (workspace: compensation motion)


Composition of actuator for compensation and motion
Lead of ball screw
1 mm
Resolution of linear scale $0.2 \mu \mathrm{~m}$

## 4. Experiments

Experiment (compensation of position error of TP)


## 4. Experiments

## Experiment (compensation of position error of TP)

Maximum values of position error

| $\theta_{y \max }\left[{ }^{\circ}\right]$ | w/o compensation |  | $1{ }^{\text {st }}$ compensation |  | $2^{\text {nd }}$ compensation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $4 x_{1}[\mathrm{~mm}]$ | $\Delta y_{1}[\mathrm{~mm}]$ | $\Delta x_{1}[\mathrm{~mm}]$ | $\Delta y_{1}[\mathrm{~mm}]$ | $4 x_{1}[\mathrm{~mm}]$ | $\Delta y_{,}[\mathrm{mm}]$ |
| 15 | 0.228 | 0.166 | 0.054 | 0.104 | 0.039 | 0.045 |
| 30 | 0.642 | 0.323 | 0.153 | 0.111 | 0.055 | 0.076 |
| 45 | 1.196 | 0.489 | 0.364 | 0.117 | 0.159 | 0.106 |
| 60 | 1.765 | 0.653 | 0.486 | 0.132 | 0.144 | 0.106 |

RMS values of position error

| $\left.\theta_{y \max } C^{\circ}\right]$ | w/o compensation |  | $1^{\text {st }}$ compensation |  | $2^{\text {nd }}$ compensation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta x_{1}[\mathrm{~mm}]$ | $\Delta y_{1}[\mathrm{~mm}]$ | $\Delta x_{1}[\mathrm{~mm}]$ | $\Delta y_{1}[\mathrm{~mm}]$ | $\Delta x_{1}[\mathrm{~mm}]$ | $\Delta y_{1}[\mathrm{~mm}]$ |
| 15 | 0.137 | 0.094 | 0.027 | 0.054 | 0.017 | 0.016 |
| 30 | 0.333 | 0.200 | 0.075 | 0.069 | 0.017 | 0.017 |
| 45 | 0.560 | 0.295 | 0.151 | 0.052 | 0.047 | 0.044 |
| 60 | 0.789 | 0.380 | 0.202 | 0.043 | 0.066 | 0.039 |

## 5. Conclusions-RPMFAP

To develop a precise rotational parallel mechanism with two DOF, structural and dimensional syntheses of a mechanism with two actuators for generating rotational output motion and two actuators for compensating for position error of the target point were carried out. Our conclusions are summarized as follows.
(1) A kinematic structure of a mechanism with three connecting chains was proposed.
(2) Dimensional synthesis of the mechanism proposed in (1) was carried out, taking into consideration singular configurations and rotational output motion coupled with translational motion for compensation.
(3) A large orientation workspace, with an output link inclination angle of up to $60^{\circ}$ and compensation for position error of the target point was successfully achieved by our prototype mechanism.

## Table of contents

1. Introduction
2. Kinematic design of parallel manipulator with redundant actuators: spatial six-dof parallel manipulator with redundant actuators for gross and fine motions
3. Kinematic design of lower-dof parallel mechanism to minimize the uncompensatable error: 3-URU pure rotational parallel mechanism
4. Kinematic design of lower-dof parallel mechanism with dof for compensating uncompensatable error
$\checkmark$ translational parallel manipulator with fine adjustment of platform orientation
$\checkmark$ two-dof rotational parallel mechanism with compensation for position error
$\checkmark$ rotational parallel mechanism with variable target point
5. Summary (Conclusions and future works)

## Table of contents-RPMVTP

1. Introduction
$\checkmark$ Background
$\checkmark$ Purpose
2. Basic structure of RPMVTP

Mechanism configuration
$\checkmark$ Velocity relationship
3. Structure of fully decoupled RPMVTP
$\checkmark$ Kinematic structure
$\checkmark$ Inverse displacement analysis
4. Numerical example
5. Conclusions
6. Future work

## Target applications



Minimum invasive surgery

(a) Convex surface ( $\mathrm{L}>\ell_{i}$ )
(b) Mature surface $\left(\mathrm{L}<\ell_{i}\right)$

Contact point $\mathrm{Q}_{i}\left(x_{\mathrm{Q}, i}, y_{\mathrm{Q}, i}\right)=$ Target point $\mathrm{C}_{i}\left(x_{\mathrm{C}, i}, y_{\mathrm{C}, i}\right)+$ Curvature radius $r_{i}$ $\frac{\downarrow}{\text { Control input }} \stackrel{\downarrow}{\downarrow}$ Tool length $\ell_{i}$

Manufacturing of prostheses

There are a lot of operations done by robots that require precise rotational output motion around two axes while the position of the rotation center (hereafter, we call "target point") being changed in a three dimensional space. Examples are shown above. In both cases, the actuators should be remotely located from the operation area in order that they would be protected from the working environment.
We are under development of the parallel mechanism with asymmetrical structure for such applications (2R3T output motion with 5 dof).

## Previous works

## Rotational/Spherical mechanisms with variable TP



Four-dof 3R1T parallel mechanism (Zlatanov and Gosselin, 2001)


Four-dof 2R2T parallel mechanism
(Okamura, Hanagasaki, Takeda, 2011)

Examples of 4 DOF mechanism

## Previous works

## position-orientation decoupled mechanism with 6 dof

Symmetrical structures:
$\checkmark$ C. Innocenti, V. Parenti-Castelli, 1991.
$\checkmark$ K. Wohlhart , 1994.
$\checkmark$ S. P. Patarinski, M. Uchiyama, 1995.
$\checkmark$ K. Mianovski, 1998.
$\checkmark$ Y. Takeda, et al, 2005.
$\checkmark$ Y. Jin, I-M. Chen, G. Yang, 2006.


Jin, Chen and Yang, 2006.


Zlatanov, et al, 1992

## Previous works

## Asymmetrical decoupled mechanism with limited dof

$\checkmark$ S. Refaat, J.M.Herve, 2006. (1R2T and 2R1T mechanisms with 3 dof have been proposed based on Lie-group theory)
$\checkmark$ J. Okamura, S. Hanagasaki, Y. Takeda, 2011 (2R2T mechanism with 4 dof have been developed) $\checkmark$ C.C.Lee, J.M.Herve, 2012. (1R3T and 2R3T mechanisms with 4 or 5 dof have been introduced, where rotational motions are infinitesimal.)


Previous mechanism by the authors' group. Actuators for translational motion are located on the output link.


Kinematic structures of mechanism in which all actuators are located on or close to the base are clarified in this research.

## Purpose of the research

$>$ To figure out kinematic structures of asymmetrical five-DOF fully decoupled parallel mechanism, in which rotational motion of the output link around two axes is controlled by two inputs while translational motion of the target point, the center of rotation of the output link, is controlled by the other three inputs.
$>$ To derive equations for displacement and velocity analyses and to clarify basic characteristics such as input-output relationship and singularity for a concrete structure.

## Table of contents

1. Introduction
$\checkmark$ Background
$\checkmark$ Purpose
2. Basic structure of RPMVTP

Mechanism configuration
$\checkmark$ Velocity relationship
3. Structure of fully decoupled RPMVTP
$\checkmark$ Kinematic structure
$\checkmark$ Inverse displacement analysis
4. Numerical example
5. Conclusions
6. Future work

## Basic structure of RPMVTP| ${ }^{1}$

## Mechanism configuration



RMC (Rotational motion generating chain): 6 dof
TPC (Target point controlling chain): 5 dof (3T2R)

## Basic structure of RPMVTP| ${ }^{\text {A }}$

## Velocity relationship (Jacobian matrix)



## Basic structure of RPMVTP ${ }^{\text {® }}$

## Velocity relationship (forward velocity analysis)



It is known from the equation that input motion of RMC generates pure rotational motion while input motion of TPC generates rotational motion coupled with translational motion of the output link. This means that rotational output motion is decoupled from translational motion in RPMVTP regardless of the kinematics structure of RMC. However, $J_{\mathrm{B}}$ is not zero matrix.

## Table of contents

1. Introduction
$\checkmark$ Background
$\checkmark$ Purpose
2. Basic structure of RPMVTP

Mechanism configuration
$\checkmark$ Velocity relationship
3. Structure of fully decoupled RPMVTP
$\checkmark$ Kinematic structure
$\checkmark$ Inverse displacement analysis
4. Numerical example
5. Conclusions
6. Future work

## Structure of Fully decoupled RPMVIW

## kinematic structure of RMC



## Structure of Fully decoupled RPMVI兩

## Possible kinematic structures for RMC

| structure | structure | structure |
| :---: | :---: | :---: |
| $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \dot{\mathrm{R}} \dot{\mathrm{R}} \overline{\mathrm{RR}}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \dot{\mathrm{R}} \overline{\mathrm{RR}} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \dot{\mathrm{R}} \mathrm{P} \overline{\mathrm{RR}}$ |
| $\mathrm{R}_{\mathrm{a}} \dot{R} \dot{R} \dot{R} \overline{\mathrm{R}} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{R} \dot{\mathrm{R}} \mathrm{P} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{R} P \mathrm{PR} \overline{\mathrm{R}}$ |
| $\mathrm{Ra}_{\mathrm{a}} \dot{R} P \dot{R} \dot{R} \overline{\mathrm{R}}$ | $\mathrm{Ra}_{\mathrm{a}} \mathrm{PR} \dot{R} \dot{R} \dot{R}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{P} \mathrm{P}$ |
| $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \dot{\mathrm{R}} \mathrm{P} \overline{\mathrm{R}} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{PR} \overline{\mathrm{R}} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \mathrm{PR} \dot{R} \dot{R} \overline{\mathrm{R}} \mathrm{P}$ |
| $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \dot{\mathrm{R}} \mathrm{PP} \overline{\mathrm{R}}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{PR} \dot{P} \overline{\mathrm{R}}$ | $\mathrm{R}_{\mathrm{a}} \mathrm{PR} \dot{R} \dot{R} P \overline{\mathrm{R}}$ |
| $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{P} P \dot{\mathrm{R}} \overline{\mathrm{R}}$ | $\mathrm{R}_{\mathrm{a}} \mathrm{PPRR} \dot{R} \overline{\mathrm{R}}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \overline{\mathrm{R}} \mathrm{PPP}$ |
| $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{P} \overline{\mathrm{R}} \mathrm{P} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \mathrm{PR} \overline{\mathrm{R}} \mathrm{P}$ | $\mathrm{Ra}_{\mathrm{a}} \dot{R} P P \overline{\mathrm{R}} \mathrm{P}$ |
| $\mathrm{R}_{\mathrm{a}} \mathrm{PR}$ P $\overline{\mathrm{R}} \mathrm{P}$ | $\mathrm{R}_{\mathrm{a}} \dot{\mathrm{R}} \mathrm{PPP} \overline{\mathrm{R}}$ | $\mathrm{Ra}_{\mathrm{a}} \mathrm{P} \dot{\mathrm{R}} \mathrm{P} \dot{\mathrm{R}} \overline{\mathrm{R}}$ |

( $\mathrm{R}_{\mathrm{a}}$ : active revolute joint on the base)
Starting from the kinematic structures for translational parallel mechanism (TPM) with three serial connecting chains(Kim\&Chung, 2003, Kong\&Gosselin, 2004, Tanabe\&Takeda, 2010), the kinematic structures shown above have been figured out as kinematic structures of RMC for fully decoupled SPMVTP. These have been obtained by adding a revolute joint at the base to the kinematic chains for TPM so that conditions for connecting chain of TPM are satisfied even when the added revolute joint is arbitrarily positioned.

## Structure of Fully decoupled RPMVI

## An example kinematic structure of RMC



Kinematic conditions(RMC):

1. The axes of the revolute joints $\mathbf{w}_{2, i}$ to $\mathbf{w}_{4, i}$ should be parallel.
2. The axes of the revolute joints $\mathbf{w}_{5, i}$ and $\mathbf{w}_{6, i}$ should be parallel while $\mathbf{w}_{4, i}$ and $\mathbf{w}_{5, i}$ should not be parallel.
3. The axes of the revolute joints $\mathbf{w}_{1, i}$ and $\mathbf{w}_{2, i}$ should not be parallel to avoid architectural singularity of RMC.
4. Rank of the Jacobian matrix with respect to RMC as a serial chain should be 6.
5. Rank of the sub-matrix $J_{\mathrm{A}}$ should be 3.

## Structure of Fully decoupled RPMVI

## An example kinematic structure of RMC


$\operatorname{PPPR}_{1} \mathrm{R}_{1}-2(\mathrm{R} \dot{R} \dot{R} \dot{R} \dot{R} \bar{R})$ mechanism
The Jacobian matrix $J_{\mathrm{T}}$ is independent of the position of the target point $P$ while it is dependent only on the orientation of the output link.


$$
\mathbf{w}_{2, i} / / \mathbf{w}_{3, i} / / \mathbf{w}_{4, i}
$$

$$
\mathbf{w}_{5, i} / / \mathbf{w}_{6, i}
$$

: active joint
O: revolute joint
: prismatic joint
$\hat{\boldsymbol{m}}_{i}$ : constraint moment of RMC-i by fixing the active joint

Velocity relationship:

## Structure of Fully decoupled RPMV年 <br> Tokyo Institute of Technology <br> Mechanical Systems Design Lab.

## An example kinematic structure of RMC: kinematic constants



## Structure of Fully decoupled RPMVI+

## Inverse displacement analysis



Kinematic structure of RPMVTP and output orientation
In total, 8 real solutions exist for inverse displacement analysis of the mechanism.

Transformation matrix:

$$
T_{\mathrm{P}}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
p & \boldsymbol{e}_{x} & \boldsymbol{e}_{y} & \boldsymbol{e}_{z}
\end{array}\right]
$$

Output pose: 5 out of
$\left(X_{\mathrm{P}}, Y_{\mathrm{P}}, Z_{\mathrm{P}}, \theta_{y}, \theta_{z}, \psi\right)$
are independent.
Procedure:

1. From $\boldsymbol{e}_{z}, \theta_{4,1}$ and $\theta_{5,1}$ are calculated.
2. From $\boldsymbol{p}, q_{j, 1}(j=1,2,3)$ are calculated.
3. Matrix $T_{\mathrm{P}, \mathrm{TPC}}$ is fully determined.
4. From $T_{\mathrm{P}, \mathrm{TPC}}=T_{\mathrm{P}}=T_{\mathrm{P}, \mathrm{RMC}, i}$, $\theta_{1, i}(i=2,3)$ are obtained.

## Table of contents

1. Introduction
$\checkmark$ Background
$\checkmark$ Purpose
2. Basic structure of RPMVTP

Mechanism configuration
$\checkmark$ Velocity relationship
3. Structure of fully decoupled RPMVTP
$\checkmark$ Kinematic structure
$\checkmark$ Inverse displacement analysis
4. Numerical example
5. Conclusions
6. Future work

## Numerical example

Kinematic constants of the mechanism used in simulation

| Symbols | Values | Symbols | Values | Symbols | Values |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $l_{\mathrm{P}}$ | 6.84 mm | $\beta_{\mathrm{B}, 3}$ | $135^{\circ}$ | $d_{1, i}$ | 50 mm |
| $b_{1}$ | 60 mm | $\beta_{\mathrm{P}, 2}$ | $45^{\circ}$ | $a_{2, i}$ | 50 mm |
| $\alpha_{3,1}$ | $90^{\circ}$ | $\beta_{\mathrm{P}, 3}$ | $225^{\circ}$ | $a_{3, i}$ | 60 mm |
| $d_{4,1}$ | 60 mm | $b_{i}$ | 110 mm | $a_{4, i}$ | 10 mm |
| $\alpha_{4,1}$ | $90^{\circ}$ | $\alpha_{0, i}$ | $45^{\circ}$ | $a_{5, i}$ | 40 mm |
| $\gamma_{1}$ | $20^{\circ}$ | $\theta_{0, i}$ | $270^{\circ}$ | $\gamma_{i}$ | $30^{\circ}$ |
| $\beta_{\mathrm{B}, 2}$ | $135^{\circ}$ | $a_{1, i}$ | 0 mm | $r_{i}$ | 35 mm |

Output motions:

1. $\left(X_{\mathrm{p}}, Y_{\mathrm{p}}, Z_{\mathrm{p}}\right)=(0,0,130)[\mathrm{mm}], \theta_{y}=30^{\circ}, \theta_{z}=\left[0: 360^{\circ}\right]$
2. $X_{\mathrm{P}}=[-20: 20][\mathrm{mm}],\left(Y_{\mathrm{P}}, Z_{\mathrm{P}}\right)=(0,130)[\mathrm{mm}], \theta_{y}=30^{\circ}, \theta_{z}=0$

## Numerical example

Case 1: $\left(X_{\mathrm{P}}, Y_{\mathrm{P}}, Z_{\mathrm{P}}\right)=(0,0,130)[\mathrm{mm}], \theta_{y}=30^{\circ}, \theta_{z}=\left[0: 360^{\circ}\right]$


Pure rotational output motion while keeping the target point at a fixed position is achieved by input motions of RMCs without encountering singular point.


## Numerical example

Case 2: $X_{\mathrm{P}}=[-20: 20][\mathrm{mm}],\left(Y_{\mathrm{P}}, Z_{\mathrm{P}}\right)=(0,130)[\mathrm{mm}], \theta_{y}=30^{\circ}, \theta_{z}=0$


Translational output motion is achieved by input motion of TPC while keeping $J_{\mathrm{A}}$ a constant matrix (as well as det $J_{\mathrm{A}}$ ).


## Conclusions-RPMVTP

In the present paper, asymmetrical five-DOF fully-decoupled parallel mechanism has been figured out and its kinematic study has been carried out.
Our conclusions are summarized as follows.

1. 24 kinematic structures of fully-decoupled asymmetrical spherical parallel mechanism with variable target point, which is composed of a target point controlling chain and two rotational motion generating chains, have been figured out.
2. Taking one structure among the 24 structures in 1, a procedure for inverse displacement analysis has been clarified. It has been also clarified that input-output relationship in velocity is independent of the position of the target point.
3. Effectiveness of the mechanism has been confirmed through a numerical example of inverse displacement analysis with a check of singularity.

## Conclusions

Concept of "Compensatability" of parallel manipulators:

1. Redundancy (combination of gross and fine motions)
2. Decoupled structure for simplicity
3. Optimal design based on Sensitivity

A family of "Compensatable Parallel Manipulators":

1. A parallel manipulator with redundant actuators for gross and fine motions
2. A 3-URU pure rotational parallel manipulator with large workspace and less sensitivity to position error
3. A translational parallel manipulator with fine adjustment of orientation error
4. A rotational parallel manipulator with fine adjustment of position error
5. A rotational parallel manipulator with variable target point

Methods:

1. Structural synthesis
2. Dimensional synthesis
3. Control

Validation:

1. Design, Prototyping and Test
2. Ikeda, T., et al, Spherical parallel mechanism with variable target point, Proceedings of 6th International Workshop on Computational Kinematics, May. 2013.
3. Okamura, J., et al, Kinematic synthesis of two-dof rotational parallel mechanism with compensation for position error, Proceedings of 2nd IFToMM International Symposium on Robotics and Mechatronics, Nov. 2011.
4. Huda, S., et al, Kinematic design of 3-URU pure rotational parallel mechanism to perform precise motion within a large workspace, Meccanica, Springer, Vol. 46, No. 1, Jan. 2011.
5. Tanabe, M and Takeda, Y., Kinematic design of a translational parallel manipulator with fine adjustment of platform orientation, Advances in Mechanical Engineering, Hindawi Publishing Corporation, Volume 2010, Dec. 2009.
6. Takeda, Y., et al, A spatial six-dof parallel manipulator with redundant actuators for gross and fine motions, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 4, No. 2, pp. 444-456, Mar. 2010.
7. Huda, S. and Takeda, Y., Kinematic design of 3-URU pure rotational parallel mechanism with consideration of uncompensatable error, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 2, No. 5, pp. 874-886, Sep. 2008.
